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PHYSICAL LABORATORY MANUAL

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PHYSICAL LABORATORY MANUAL

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CAMBRIDGE
HARVARD UNIVERSITY PRESS
1920

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PREFACE

THIS manual has been adapted particularly to the present equipment and requirements of a course in experimental physics in Harvard College known as Physics C. This course is accompanied by a series of illustrated lectures to the class as a whole, two each week, and one hour each week of recitation and problem work to the class in small sections. A knowledge of algebra and plane geometry, and a slight acquaintance with the notation of trigonometry are necessary. Familiarity with the use of logarithms or the slide rule is strongly recommended for reducing the labor of computation. The course should properly be preceded by a more elementary course in physics.

Small type has been used for a few descriptive and explanatory sections of the text which are not immediate parts of the specific directions of the experiments. The small type does not, however, signify relative unimportance of the material. Introductory remarks setting forth the application of the principles involved precede each experiment or each group of experiments involving the same principles.

In writing this manual, some material, for which much indebtedness is expressed, has been freely taken from Professor Sabine's *Laboratory Course in Physical Measurements.*

E. L. C.

PREFACE TO THE REVISED EDITION

For two reasons a revision of this book has become necessary. The distribution of time in Physics C has been so altered that the laboratory work now occupies a shorter time than formerly, and the exercises have therefore become unduly long. It is also desirable to include a wider variety of experiments in the manual, so as to fit the somewhat different needs of the various classes of students who use it. In the course of the revision many experiments have been altered, where experience has shown this to be advantageous, or on account of changes in the apparatus. The experiments which have been added to the course have all been tested out in practice, and it is hoped that most of their defects have in this way been discovered, and eliminated.

Acknowledgment is gratefully made for valuable suggestions from many friendly critics, but special thanks are due to Mr. James A. Duncan, Assistant in Physics, for his help in the preparation of this revision.

E. L. C.
F. A. S.

SEPTEMBER, 1920.

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PHYSICAL LABORATORY MANUAL

INTRODUCTION

The purpose of the laboratory work in Physics C is not only to teach physics but to develop in the student the ability to experiment. The latter implies a great many qualities and includes the ability to grasp the object and method of the experiment, the skilful manipulation of the apparatus, the power of quick and accurate observation, the capacity to derive results and conclusions, and, by no means least in importance, the ability to make a neat, concise, and complete record of the experiment. The characteristics which make a good experimenter are exceedingly valuable not only to those who may wish to continue in the study of an experimental science but to those who follow other lines of work.

The experiments of the course vary greatly in character, but they are chosen so far as possible to be equally interesting and beneficial to all students. Some experiments may be the same in principle as experiments already performed in elementary physics, but, in these cases, it is the object to obtain more accurate results through the use of more precise methods and better apparatus. Other experiments are of value more because of the principles which they teach. There are several experiments in the course which are distinctly engineering problems, but they are interesting because of the commercial application of some of the physical principles.

The manual is not intended alone to supply the necessary text in preparation for the laboratory work, but is intended merely as a guide in indicating the object and method of the experiment. Satisfactory results cannot be obtained in the laboratory unless outside work is done in preparation for the experiments. The textbook should be studied on the principles involved, and, if necessary, other books should be consulted.

GENERAL DIRECTIONS

The instructor will refer to the appropriate numbered paragraph in the following general directions as "G.D. No . . . ," for brevity, in criticizing reports.

1. *Assignments.*

Each experiment is to be performed on the day assigned. An experiment performed on any other day, without the permission of the instructor, may be given no credit.

2. *Data.*

The original data are to be taken down as neatly and systematically as possible, on approved laboratory paper. This "data sheet" is to form part of the final report on the experiment, and should include every observation, as originally taken. If, for instance, the difference of two quantities is to be measured, and the values of each are first obtained separately, both sets of values, as well as their differences, should be entered. The data sheet is to be shown to the instructor at the close of the laboratory period for examination, and stamping, and must not be altered in any way thereafter.

3. *Units.*

Every numerical reading should be accompanied by the units employed, excepting that, in tabular forms, one statement of the unit is sufficient. In the metric system, only one unit is to be used: for instance, a length of 4.26 cm. must not be recorded as 4 cm. 2.6 mm.

4. *Estimation to tenths.*

Every reading, unless there are directions to the contrary, must be made to the smallest possible fraction of a scale division, *e. g.* tenths of mm. on a metric scale; and these must be recorded, even if they are zeros. Estimation to halves, thirds, or quarters is not usually sufficiently accurate.

5. *Reports.*

A report is to be written on each experiment. The value of experimental work depends largely upon the way in which it is recorded. The record should be so clear and so well arranged that another person, say a fellow student, could understand it without reference to any other sources.

6. *Paper.*

Reports are to be written on special paper, protected by a cover, and furnished with a title-page. Reports must be written in ink, or typewritten; but diagrams and graphs may be in pencil.

7. *Form of Report.*

The following order is recommended:

(*a*) The object of the experiment.

(*b*) A list of the main pieces of apparatus used, together with laboratory numbers thereon, if any.

(*c*) A concise but clear statement of the theory and method of procedure of the experiment. This section must be written in the student's own words; it is useless to copy out the laboratory directions. Actual manipulations may be indicated very briefly, but time and care should be spent on making the purpose of each part of the work very clear. Diagrams will often be of great assistance, but they should not be perspective drawings.

(*d*) The data sheet. See G. D. No. 2.

(*e*) Discussion of the data; derivation of results; discussion of the sources of error, etc.

(*f*) Answers to questions in the directions, if any.

(*g*) Graphs, if any.

8. *When Due.*

The finished report is due at the beginning of the next laboratory exercise after the day of the performance of the experiment. Reports one week late will, in general, be given only one half credit. Reports more than one week overdue may be given no credit.

The reports will be examined and, if necessary, handed back for correction, in which case they are due at the next succeeding laboratory exercise.

9. *Working in Pairs.*

The entire experimental work and the report are to be the work of the student whose name appears on the cover, unless more than one student work with the same apparatus, in which case the names of the co-workers should be recorded. In all cases the final report is to be the work of the individual student.

10. *Quality of Reports:*

The quality of a student's work will be judged rather by the evidence of time and thought spent upon the report than by accurate numerical results, although accuracy is, to some extent, a measure of the student's ability. The standing of a student will also depend upon his knowledge of the experiment as shown by answers to questions which may be asked him at any time in the laboratory, upon the observance of the rules for significant figures, upon the way in which he handles the apparatus, and upon the way in which the apparatus is left.

11. *Reliability of Results.*

Every direct measurement of a physical quantity, and every result derived from any number of observations or measurements have a certain reliability dependent upon the following factors: the method of observation, the care exercised in manipulation, the nicety of the apparatus, etc. In other words, to every result there is assignable a certain definite accuracy which is sometimes difficult to determine, but which, in most cases, can be approximately found, as will be explained below. By the proper use of significant figures the accuracy of a result can be clearly indicated, and, conversely, a misuse of significant figures is very misleading and may inspire greater reliance upon a result than the work justifies.

12. *Significant Figures.*

Significant figures are those figures in a number or decimal which remain after all ciphers, necessary only to locate the decimal point, are removed. For instance, in 25,000 and .0025, the 2 and the 5 are significant. In some cases a cipher may be significant as, for instance, in the expression of the length of an object as .001250 cm. The final cipher may here be necessary in order to show that the measurements were made to that figure, but the result happened to come out as an even 0.

As an example to illustrate the way in which the proper use of significant figures can indicate the accuracy of a measurement, consider the following. If, in weighing with balances sufficiently sensitive to detect milligrams, the weight of a body is exactly twelve grams, it should be entered as 12.000 grams. The three zeros, in this case, are significant and are necessary to express the full accuracy of the observation. As another example to illustrate the use of significant figures, suppose the volume of a certain body, as found by calculation from its dimensions, comes out as 2463.072 c.c., whereas from considerations given below it is known that this volume is doubtful in the place occupied by the 6. All of the succeeding figures are of no use and must be discarded, keeping only three significant figures. The result will be written $24\underline{6}0$ c.c., and in order to show that, in this case, the final zero is not significant, the last significant figure, which is the doubtful figure, is underlined. This practice of underlining the doubtful figure is exceedingly helpful, and will be used in this course.

The method of determining the approximate accuracy of a result will be illustrated by the following example. The measurement of the dimensions of a cylinder by a vernier gauge gives as its length $5.21\underline{5}$ cm., and as its diameter $3.02\underline{0}$ cm. The vernier gauge cannot accurately measure thousandths of a centimeter, and the last figure in each of the above values is in great doubt. The true length and the true diameter of a cylinder may be either greater or less than these values by .002 cm. To calculate the volume of the cylinder we have the formula $V = l \pi r^2$. The radius is $1.51\underline{0}$ cm. The arithmetical work is given in full, as it is most instructive: —

$$\begin{array}{r}
1.51\underline{0}\\
1.51\underline{0}\\
\hline
\underline{0.000}\\
15.1\underline{0}\\
755.\underline{0}\\
151\underline{0}\\
\hline
2.28\underline{0100}
\end{array}$$

Since the last figure in the value of the radius, the cipher, is in doubt, all of the figures underlined are in doubt. The first cipher in the product is in doubt, therefore the succeeding figures are wholly unknown, and should be discarded. Proceeding with the calculation,

$$\begin{array}{r}
2.28\underline{0}\\
3.141\\
\hline
228\underline{0}\\
912\underline{0}\\
228\underline{0}\\
684\underline{0}\\
\hline
7.161\underline{480}
\end{array}$$

Again discarding the figures which are wholly unknown and keeping only the first figure which is in doubt,

$$\begin{array}{r}
7.16\underline{1}\\
5.21\underline{5}\\
\hline
\underline{35805}\\
716\underline{1}\\
1432\underline{2}\\
3580\underline{5}\\
\hline
37.3\underline{44615}
\end{array}$$

The final result is to be entered as 37.3$\underline{4}$, the succeeding figures, which are wholly unknown, being discarded. That there has been a great saving of labor will be obvious, if the student will but repeat the above calculations without discarding any of the figures.

13. *Errors as Percentages.*

A better method of discussing the results, and one which also applies to logarithms, is given below. If the calculation involves only multiplication and division, the percentage error introduced into the result by any factor is equal to the percentage error in that factor. Thus, in the example above, the error which could pass undetected in the measurement of the length is supposed to be .002 cm.; this is .04 per cent of 5.21$\underline{5}$. Therefore, the possible error in the result arising from this source is .04 per cent of 37.3446 . . . or .014 c.c. Obviously, therefore, the first figure 4 in the result is slightly in doubt, and the subsequent figures are unknown and must be discarded. If one of the factors is squared, the percentage error in the result is twice as great. If a factor is cubed, its error is tripled in the result. If the square root of a factor is taken, the possible error is halved in the result. Thus, in the above calculation of the volume of the cylinder, the radius is squared. The error in the measurement of the diameter which could pass undetected is assumed to be .002 cm. This is nearly .07 per cent of 3.02$\underline{0}$, — the diameter. Since the radius is squared, the possible percentage error arising in the result from this source is .14 per cent; .14 per cent of 37.3446 c.c. is .05 c.c. Thus, the first figure *four* in the result is in doubt, subsequent figures are unknown, and the result should be written 37.3$\underline{4}$.

The above discussion has shown how the accuracy of a result can be approximately determined when the errors in the several factors are known. How then can the accuracy of the factors themselves be improved?

14. *Classes of Errors.*

There are several kinds of errors which may affect an observation. One class of errors called *instrumental errors* arise from defects in the apparatus, and can only be detected and diminished by changing the conditions of the experiment or by using different apparatus. An observation is subject to another kind of error if the experimenter persistently makes the measurement too large or too small, as might be the case, for instance, in judging the instant a pendulum swings by a certain reference mark at the lowest point of its swing. This type of error may be classified as errors due to *personal equation.* The third kind of error is due to the expected chance mistakes in observations, and is just as likely to be above as below the true value of the factor. It is this kind of error which can be materially diminished by using the average of several observations of the same quantity.

If several equally careful observations have been made of the same quantity, the mean of them all is what Holman calls the "best representative measure." The error of this mean of course depends upon the errors in the individual observations, but the mean is more accurate than a single observation in proportion to the *square root* of the number of observations which are averaged.[1]

15. *Relative Importance of Errors.*

It is well to note that, although the final result is desired to the greatest possible accuracy, it does not follow that every factor is to be determined with the greatest care. From the discussion previously given, each factor introduces into the final result an error proportional to the error in the factor, so that a result cannot be more accurate than the least accurate factor. It is, therefore, time wasted to determine one factor with extreme care when the inevitably large error in another factor will make the result so inaccurate as to completely mask the care taken in the measurement of the first factor. The time should be spent on the determination of the least accurate factor.

---

[1] A very clear discussion of the subject of Precision of Measurements may be found in Holman's "Discussion of the Precision of Measurements," or in Goodwin's "Precision of Measurements and Graphical Methods."

16. *Graphical Methods.*

When a series of results depends upon one regularly varying factor in the experiment, a very instructive graphical representation may be made by plotting the results on coördinate paper. When this is done, an examination of the graph shows, in a much clearer manner than does a column of figures, the way in which the results vary as the factor upon which the results depend changes. This method of recording results may be most simply illustrated by an experiment in which the temperature of a cooling body has been observed at frequent intervals; in this the temperature observations form the series of results depending upon the time which has elapsed as the one varying condition. In the following the first column of figures gives the time in minutes which had elapsed from the beginning of the experiment, and the second column the corresponding temperature observations: —

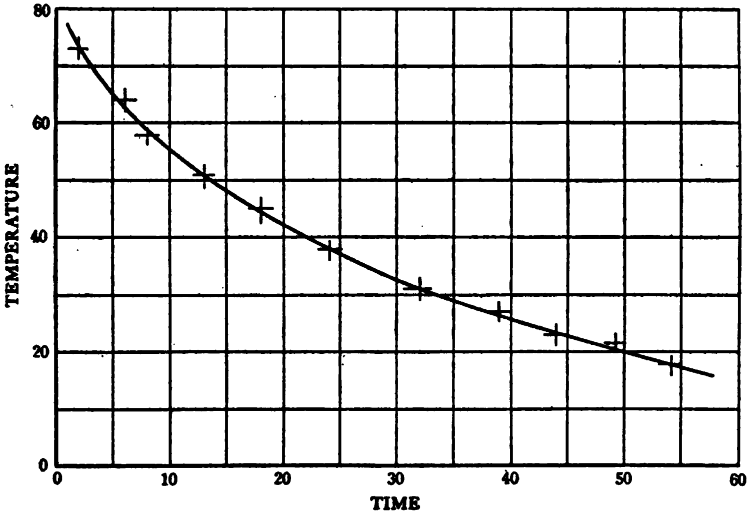

FIG. 1

Time	Temperature	Time	Temperature
2 min.	73° C	13 min.	51° C
6	64°	18	45°
8	58°	24	38°

On the coördinate paper, which is ruled by numerous equally spaced lines, a certain horizontal line is chosen as the *axis of* *abscissae*, and a certain vertical line as the *axis of ordinates*; the intersection of the two lines is called the *origin*. A horizontal distance measured along the axis of abscissae is to represent the time from the beginning of the experiment to a certain observation, and a vertical distance measured from the point thus reached is to represent the corresponding temperature observation, each according to some chosen scale. Each observation is to be plotted in this way and a curve drawn through the resulting points. In the curve plotted in Fig. 1, each horizontal space represents 5 minutes, and each vertical space represents a change in temperature of 10°.

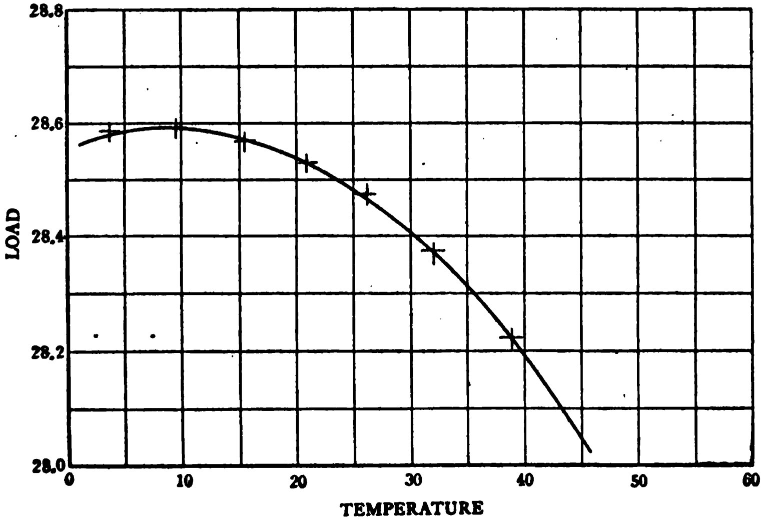

FIG. 2

17. *Smooth Curves.*

It is not always desirable, or possible, to draw a curve through all of the points marked, but if it is known that the curve should be a smooth curve, and if the points are numerous, it should be drawn between them, thus averaging, in a measure, the errors of observations.

18. *To Show Variations in Large Quantities.*

If the quantity plotted is large in comparison with its variations, the scale on which the curve is plotted must be correspondingly large in order that it may show these variations. This may be done without using an awkwardly large sheet of paper by plotting only the curve and dispensing with the paper below. This is illustrated in Fig. 2, which is the curve representing the load required to sink a Nicholson hydrometer in water at various temperatures.

19. *Position of Origin.*

If a quantity is being plotted as ordinates, some of whose values are positive and some negative, the positive values are plotted upward from the axis of abscissae, the negative values downward. Thus, in the first example, if the body had cooled below zero, the curve would have crossed the axis of abscissae. When the abscissae are negative they are to be plotted off to the left of the origin.

20. *Choice of Scales.*

The scales for abscissae and for ordinates are to be so chosen that the observations cover a large part of the sheet of paper in both directions. At the same time, each ruled space of the paper must represent a convenient number of units, and the heavy lines must correspond to whole numbers. The two scales need not be alike.

21. *Labelling the Scales.*

The quantity represented by each scale must be plainly indicated, and also the units used. The scales must be numbered, the numbers occurring regularly but not being crowded. Numbers corresponding to the observed points themselves should not be written in.

22. *Points.*

The points on the graphs should be recorded as fine dots, made with a sharp pencil; and these should be made conspicuous by drawing fine cross lines exactly through each point, or, better still, to avoid obliterating the points themselves, by drawing circles, or other hollow figures around the points.

SECTION I. MECHANICS

INTRODUCTION TO EXPERIMENTS 1 AND 2

In these two experiments the density of some materials will be found. The value of the experiments lies not in the principles involved but in the methods and means of obtaining the results. The main point of the experiments is to obtain, through the use of accurate apparatus, and by giving attention to sources of error and careful experimental manipulation, much more accurate results than may have been obtained by the same general methods in previous work.

In performing these experiments, much thought should be given to the correct use of the apparatus, to possible sources of error, and to devising a good scheme for recording the data. The greatest possible accuracy should be obtained. These experiments, being the first of the course, give an opportunity to become acquainted with advanced experimental methods.

The *vernier* is a device which will be used in many experiments as a means of obtaining greater accuracy in measurement. In preparation for the first part of Exp. 1 the following section on the vernier should be studied.

The Vernier

The *vernier* is a device which assists in the accurate reading of a scale. A scale is used to measure the relative amount of motion of one part of an apparatus with respect to another part. The scale is usually an integral part of one of the members of the apparatus and is usually fixed, while on the other piece of the apparatus is found an index mark which moves along the scale and indicates the distance the one part has moved with respect to the other. The number of whole divisions the index mark has moved from the zero position can be read off directly, and tenths of the smallest division estimated, but the vernier enables the parts of the smallest division to be accurately and easily determined.

The vernier consists of a small subsidiary scale marked off beginning with the index mark as zero, and moves in contact with, and flush with the main scale. The n divisions of the vernier are made equal to a certain number m of the smallest divisions on the main scale. In most cases m is equal to $n - 1$; but occasionally m may be equal to $n + 1$, in which case the vernier is called *retrograde*. Each division of the vernier is, therefore, equal in length to m/n times the value of the smallest division of the main scale. The difference between the value of a main scale division and a vernier division is the *least count* of the vernier, and is the smallest scale value directly readable by the vernier.

The following examples will make the above considerations clearer. The vernier, shown enlarged in Figs. 3 and 4, would be used to indicate the tenths of millimeters on a centimeter scale. Suppose the smallest divisions of the main scale are millimeters. The n, i. e. 10, divisions of the vernier are equal to m, i. e. 9, divisions of the main scale or 9 millimeters, so that each vernier division is equal in length to m/n, i. e. 9/10, of a millimeter. The difference between this and one main scale division is 1/10 *m.m.*, which is the least count of the vernier, and is the smallest value that can be directly read by the vernier.

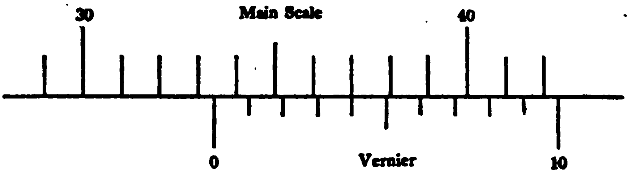

FIG. 3

If the index or zero of the vernier coincides with the zero of the main scale, no mark of the vernier except the 0 and 10 will coincide with a mark of the main scale. (See Fig. 3.) The first mark of the vernier falls short of a mark on the main scale by just 1/10 of a millimeter, the second mark by 2/10 of a millimeter, etc. If the vernier were moved, say 7/10 of a millimeter from the zero position, the seventh vernier mark would coincide with a mark on the main scale. It is therefore clear that the vernier mark which coincides with a mark on the main scale indicates the number of tenths of a millimeter the index mark has moved from the last even millimeter mark on the main scale. The reading for the setting shown in Fig. 4 is 33.4. *To read any vernier, take the reading of the scale division next preceding the zero of the vernier and add* p *times the least count of the vernier, where* p *is the division of the vernier in coincidence with a division on the main scale.*

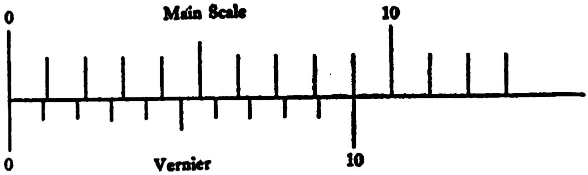

FIG. 4

Fig. 5 shows another form of vernier which will be found on the spectrometer. In order to find the least count of this vernier, it is necessary to find how much each vernier division is less than a division on the main scale. The smallest division on the main scale is 1/3 of a degree, or 20 minutes, and 60 vernier divisions are equal in length to 59 main-scale divisions, making one vernier division 59/60 of a main-scale division. The least count of the vernier is, therefore, 1/60 of 20 minutes, or 20 seconds. Three divisions on the vernier indicate one minute, and the number of minutes along the vernier are indicated by numbers.

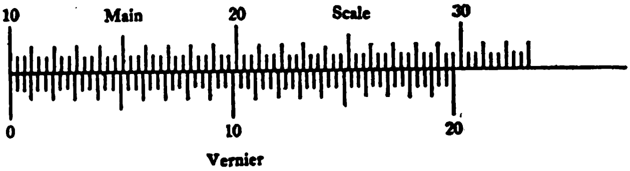

FIG. 5

I. VOLUME OF A SOLID

Read *Introduction to Experiments 1 and 2*, and the section on the *Vernier*.

Object. The objects of this experiment are to become familiar with the vernier gauge and the micrometer caliper; to find the dimensions of two specimen cylinders, and hence their volumes; and to check the values of the volumes so obtained by immersion in water.

Apparatus. Models of several types of verniers, very much enlarged for convenience, are supplied for examination and study.

A vernier gauge and micrometer caliper together with a small steel cylinder, or its equivalent, and a large aluminum cylinder, or its equivalent, are provided for the volume determinations; also graduated glass cylinders ("graduates").

THE VERNIER GAUGE

The *vernier gauge*, an instrument for the measurement of length, and shown in Fig. 6, is provided with two jaws between which is placed the object to be measured. One jaw is permanently fixed to a metric scale upon which the other jaw slides. On the movable jaw is the index mark which should coincide with the zero of the scale when the jaws are together; and when the

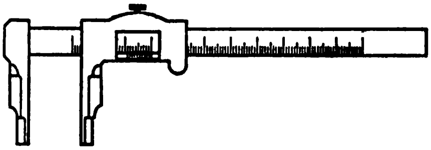

FIG. 6

object to be measured is placed between them, this line marks on the scale their distance apart, and thus indicates the length of the object. After recording the number of whole centimeters and millimeters, if a fraction of a millimeter remains, it also may be read off directly and accurately by means of the vernier. A more careful examination of the vernier will give some information also in regard to the next decimal place.

The zero error, to be applied as a correction to all measurements, should be carefully determined by taking the reading when the jaws are pressed gently together. A plus or a minus sign should be prefixed to indicate whether the correction is one to be added or subtracted. Holding the instrument to the light, note at what point the jaws touch, and use this part of the gauge in subsequent measurements.

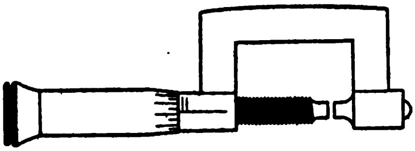

FIG. 7

THE MICROMETER CALIPER

The *micrometer caliper*, shown in Fig. 7, although not as convenient as the vernier gauge and incapable of measuring as great lengths, is, nevertheless, a more accurate instrument. It consists of a screw moving in a nut toward or away from a fixed stop. One complete turn of the screw advances or withdraws its end by an amount equal to the distance between the threads of the screw, — a fraction of a turn by a proportional amount. The number of whole turns may be read on a longitudinal scale, and the fraction of a turn by graduations upon the head of the screw. The object to be measured is placed between the stop and the end of the screw, and the latter turned down upon it. The length of the object is then read off directly in units which depend upon the pitch of the screw. The zero error is to be determined before and after measuring by taking the readings with the jaws closed. This correction, with the proper sign, must be applied to all readings. If the instrument is provided with a friction or ratchet head, it should always be taken hold of in turning down the screw, thus securing uniform pressure. If a ratchet head is being used, it should be turned until the ratchet just begins to act, but it should not be allowed to turn further, as the pounding would increase the pressure.

Procedure. Examine and study each type of vernier provided, making sure that each vernier is perfectly understood. Record a reading on each vernier, and have the readings checked by the instructor. Record the least count of the vernier used.

Measure the length and diameter of each specimen. At least five independent observations of the length of each specimen should be made, and not less than ten measurements of the diameter of each should be recorded. Since it cannot be assumed that the specimens are exactly regular in shape, the measurements should be made at various places in order that a better average dimensions may be obtained; the tips of the jaws should be used for these measurements.

Observe that in measuring a particularly small dimension a very accurate instrument is required in order that the percentage error may not exceed the percentage error made in the measurement of the other larger factors. Special care should be used in measuring the diameter of a specimen because, since it is squared in finding the volume, the error produced in the result by an error in the factor is doubled.

If water is poured into one of the graduates, the position of the surface may be read, by careful estimation of parts of a division, to an accuracy corresponding to one-tenth of a cubic centimeter. The lower part of the surface is to be used, as the rest of it is merely a thin film of water drawn up at the edges. A quantity of water should be put into the graduate sufficient to cover the specimen when immersed, and the positions of the surface are to be read before and after the specimen has been let down into the water. Splashing must be avoided by tipping the graduate while putting the specimen into it. The difference of the two readings gives, of course, the volume of the cylinder. The volumes of both cylinders are to be obtained in this way. They are also to be obtained by calculation from the measured dimensions. Record all results to the proper number of significant figures, and include in the report a brief discussion of the accuracy of the two methods of getting the volume.

Do not fail to record the identifying numbers of the specimens, as one of the same ones is to be used again. Include in the report a simple sketch, duly explained, of a vernier designed to read to the fifth part of a scale division.

2. WEIGHT AND DENSITY

Read *Introduction to Experiments 1 and 2.*

Object. The primary object of this experiment is to give practice in the use of the equal-arm balance for the very accurate measurement of the mass of a quantity of matter in terms of the standard masses called weights. All corrections, made necessary by the inequality in the arms of the balance and because of the buoyancy of the air on the weights and object, will be made. These corrections will be shown to be very considerable when the accurate weight of a body is desired.

Apparatus. A good equal-arm, analytical balance, shown in Fig. 8, and set of weights are provided for use in this experiment.

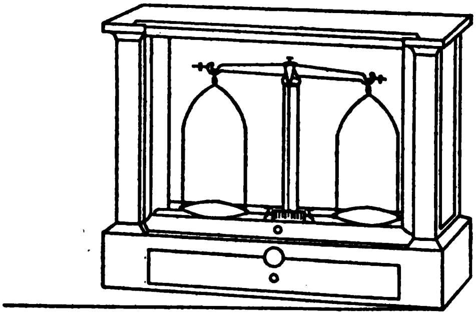

FIG. 8

One specimen cylinder, one of those used in Exp. 1, is to be weighed. If Exp. 2 is done first, any cylinder may be used, but its number is to be noted, and the same one is to be used in Exp. 1., together with a second cylinder.

THE EQUAL-ARM BALANCE

The following is a description of the method of using an equal-arm balance and the corrections necessary in determining the mass of a body. In subsequent experiments when the equal-arm balance is used, these directions must be followed.

Reading by Vibrations. Since time is consumed in waiting for the pointer of an ordinary balance to cease swinging, it is best to determine the point at which it would come to rest by observing its turning points. The mean between the average of the readings to the right, and the average of those to the left, of several consecutive swings, will be the point of rest. To correct for the error due to the diminution of the amplitude of swing by friction, an *uneven* number of swings should be taken, thus throwing both the greatest and the least upon the same side. If the pointer moves over a small scale numbered from 0 to 20, suppose the consecutive excursions are as follows:

To left	To right
8.3	
	12.6
8.9	
	12.2
9.6	
3) 26.8	2) 24.8
8.9	12.4
	8.9
	2) 21.3
	10.6 point of rest.

The balance should be so leveled on the table that it cannot rock and thus change the zero point during the progress of the experiment. Disturbing air currents, which will make the balance swing irregularly, may be avoided by always lowering the front of the case while weighing. As the pointer is some distance in front of the scale, its reading will depend somewhat upon the position of the observer. Error from this source may be avoided by placing a small mirror behind the pointer, and always taking such a position that the latter hides its own image.

The zero point is to be redetermined each day before beginning work with a balance; usually two sets of observations are sufficient.

Double Weighing. In accurate work with balances it is necessary to correct for inequality in the lengths of the arms. The ratio of the lengths may be determined and the correction applied by the following method. Let the lengths of the arms be a (left) and b (right). Place the body to be weighed in the left scale pan. By trial, determine what weight in the opposite scale pan will bring the pointer nearest to the true zero point previously obtained, and determine the point of rest by vibrations. If this is not exactly the zero, balance is not quite obtained. Add one centigram. This will perhaps send the pointer to the other side of the zero. One weight is too small and the other is too large, but the exact weight required can be found by the following process of interpolation. The difference between the two readings (the distance that the pointer is moved by one centigram) is the *sensitiveness* of the balance; its reciprocal is the fraction of a centigram required to move the pointer one division; the latter multiplied by the distance from the first point of rest to the zero point is the fraction of a centigram which should have been added to secure exact balance; for example, if the zero point is 10.5, and if the point of rest is 11.2 when the weight is 55.16 gms., and 9.8 when the weight is 55.17 gms., one centigram moves the pointer 1.4 divisions, the weight necessary to move it 1 division being $\frac{1}{1.4}$ of a centigram; and the weight necessary to move it from 11.2 to 10.5, that is, .7 of a division, is $\frac{.7}{1.4} = .5$ of a centigram, giving for the true weight necessary to secure balance $w_1 = 55.165$ gms. This method, called interpolation, makes it possible to weigh to milligrams, although the smallest weight in the set is a centigram.

From the principle of the lever we have, if x is the unknown weight,

$$ax = w_1b.$$

Change x to the opposite scale pan, and determine the weight w_2 necessary to exactly balance it. The weights now are on the arm a, and the equation becomes

$$bx = w_2a.$$

Multiply one equation by the other.

$$abx^2 = w_1w_2ab;$$

whence $$x = \sqrt{w_1w_2},$$

or $$x = \frac{w_1 + w_2}{2},$$

approximately, since w_1 and w_2 differ but slightly. Calculate x.

This process of weighing a body, first in one scale pan and then in the other, to eliminate the error arising from the inequality in the lengths of the arms of the balance, is called *double weighing*.

Weight in Vacuo. The operation of weighing consists, strictly, in finding the number of standard masses called grams which are attracted by the earth with as much force as is the body which is being weighed. If a weighing has been made in air by means of the equal-arm balance, it does not follow that the earth is attracting with equal force the weights and the body which is being weighed, even after a correction has been applied by double weighing for any accidental inequality in the lengths of the arms. The experiment is complicated by the buoyant action of the air which tends to lift not only the body which is being weighed but also the brass weights. Properly, therefore, the experiment should be performed in vacuo. It is, however, usually more convenient to work in the air and then apply the necessary corrections. The buoyant force of the air, the lifting force which it exerts, is equal to the weight of the air displaced.

It is desired to find from the result of any weighing the number of grams which would have balanced the body being weighed had the experiment been performed in vacuo. For a moment regard only the buoyant force exerted by the air on the brass weights. Had the experiment been performed in vacuo not so many weights would have been required, since they would not be lifted in part by the air. The first correction, therefore, is to subtract from the above result the weight of the air displaced by the brass. To find the value of this correction, calculate first the volume of the brass by dividing the number of grams by the number of grams required to occupy one cubic centimeter of space. Multiply this, the volume of the air displaced, by the weight of each cubic centimeter of air as determined from Table I given in the Appendix. The result is the correction to be subtracted. On the other hand, the body being weighed has also been buoyed up by the air, and consequently appears lighter than when weighed in vacuo. A second correction must therefore be applied by adding the weight of the air displaced by the body.

The result thus corrected is the number of grams which are attracted by the earth with the same force as is the body. This is the knowledge that is almost always sought in weighing; and strictly, no accurate weighing is complete until these corrections have been applied.

Procedure. Any accurate weighing involves finding the weight necessary to bring the "point of rest" of the pointer (when the weight is on) into coincidence with the "zero point" (when the weights are off). If the point of rest, when tested several times in succession, proves not to be constant, there is evidently a limit to the accuracy of the weighing that can be done with the balance. It is often worth while to determine this limit of accuracy, so that one may know just how far results may be relied upon. For this purpose, at the beginning of this experiment, the zero point is to be determined at least five times in succession, by the method of interpolation, as described above. Before each determination, raise the balance beam by means of the device in the front of the case, and then lower it again. This gives the knife-edges a chance to make contact with the surfaces on which they bear in a new way, if this is possible, and this may alter the arms of the balance very slightly, and hence the zero point. Average the five zero points, when found, and see which one differs most from the average, and how much. We may assume the average to be nearly correct, and we may call the differences from the average the "errors" of the single values of the zero points. Evidently in any weighing with this balance we may again get an error as large as the largest we have already obtained. If so, all weighings will be uncertain to such an amount as would of itself displace the pointer by the amount of this error. This small weight can be calculated from the *sensitiveness* of the balance; and it may be taken as the *limit of accuracy* of the balance; no weighings need be carried out to any higher degree of precision than this.

Carry through the process of double weighing, as described above. In a two-hour laboratory period, there will probably not be time to do the work on "weight in vacuo," but it is of great importance in accurate work, especially when weighing substances whose density is very different from that of the weights used.

Determine the sensitiveness, the limit of accuracy of the balance, the true weight of the specimen and the ratio of the lengths of the balance arms (*a* and *b* in the above theory). In all cases, each point of rest is to be found at least *twice*, and the average determined.

From the weight, and the volume as found in Exp. 1, calculate the density of the specimen. Identify the material of which it is made from a table of densities.

What percentage error would be caused in the weight if the correction for inequality of the balance arms were not applied ?

If this experiment is done before Exp. 1, the report may be held for a week, until the volume as found in Exp. 1 becomes available.

3. FALLING BODY

Object. The object of this experiment is to measure the time of fall of a small ball through a measured distance, and hence to deduce the value of g, the acceleration of gravity.

Apparatus. Special release apparatus on wall, high up; receiving platform on floor; slider on board, with smoked glass, receiving traces from a " tracing magnet," and from a tuning fork.

Method. The ball is supported on the end of a light wooden beam, which may, on closing the " trigger " circuit, be released and pulled away from under the ball, so that the latter falls freely. This action breaks an electric circuit, and causes the tracing magnet to move. The electric connections are shown in a diagram with the apparatus. The same circuit is closed again automatically, in time to be broken once more by the arrival of the ball at the receiving platform. The motions of the tracing magnet produce a line on the smoked glass which is fastened to the slider, if the latter is drawn through by hand. The "trigger" key closes as soon as the end of the slider reaches it and starts the fall at the proper moment. The tuning fork is automatically maintained, and controlled by a switch; it must be started just before a record is taken, and turned off just after.

The record itself consists of a wavy line produced by the fork, with another line, broken in places, beside it, made by the tracing magnet. These two lines should be close together. The time of fall can be read off this record in terms of fork vibrations, which must be carefully counted, marking every ten with a pin, to avoid getting lost on the way. The tracing magnet starts from its steady position and bumps against a stop. This bump furnishes a sharp and definite mark on the record, which is easy to fix with precision; it is to be taken as the true time of start; likewise for the bottom signal. These times must be estimated as closely as possible, to a small fraction of a fork vibration.

The experiment calls for at least two successful records of the fall; the slider is to be run through by hand, as steadily as possible, and at such a rate that it takes about one second for the whole operation. A little preliminary practice is recommended, to get a fairly uniform motion of about the right speed.

In order to translate these time records into seconds, we must find the rate of the fork. For this purpose, turn the two-way switch over to " clock," and the tracing magnet will then give a signal every second. Then obtain two successful records of the fork against the clock, but during these, move the slider somewhat less fast, so that each record covers two consecutive seconds. Count, the number of vibrations in the two-second interval, and hence get the rate of the fork. From this follows the actual time of fall. Measure the actual distance of fall with a metric tape, and deduce the value of g.

In the report on this experiment a brief description of the apparatus is sufficient. Discuss carefully the sources of error. Show which needs the more careful attention, the length or the time, for accuracy in the final result. Give reasons showing what would be a fair estimate of the attainable accuracy in each of these quantities, and in the final result. Present these estimates in the form of "percentage errors."

4. SPEED OF A BULLET

The object of this experiment is to measure the speed of two types of rifle bullets. The bullet is shot into the bob of a heavy pendulum which is initially at rest. The resulting motion of the bob is measured; the momentum of the bob immediately after the impact is deduced, and hence the momentum and the velocity of the bullet just before.

The apparatus consists of a pendulum hung by four wires, the bob in the form of a heavy cylinder, set horizontally. The rifle is mounted horizontally, so as to fire into one end of this cylinder, along its axis. The inside of the cylinder is filled with a wooden plug which receives the bullet. There is a frame between the rifle and the pendulum which holds a paper diaphragm to shield the pendulum from the blast of air from the rifle. The rifle should be at least three feet from the bob, and the screen about mid-way. An aluminium indicator is moved along a scale by the motion of the bob, and this makes it easy to find the distance D through which the bob moves. This distance should be found by two or three trials for each size of bullet. The vertical distance L from the middle of the cylinder to the level of the top of the supporting wires is also to be measured. The pendulum is to be detached from the wires at the end of the measurements, and weighed; also a bullet is to be extracted from each sort of cartridge and weighed to an accuracy of at least a hundredth of a gram.

From L and D find h, the distance the center of gravity of the bob rises (vertically) due to the impact. All the energy involved in the velocity V of the bob after impact is spent in raising the bob during the swing D. Hence the relation between V and h is the same as if the pendulum had moved up the distance h vertically. Find V.

From these quantities find the momentum of the bob after, and of the bullet before the impact; and hence find the velocity of each bullet. In this calculation, the fact that the pendulum gains mass by taking in the bullet may be ignored, as the bullet is comparatively light.

Measure also the length of the barrel, and the diameter of the bullet. Find, from the energy of the bullet, the average force acting on the bullet while it was inside the barrel, and hence find the average force per sq. cm. or the pressure in the barrel during the explosion.

Should the kinetic energy of the bullet just before the impact be equal to that of the bob just afterwards? Why?

INTRODUCTION TO EXPERIMENTS 5, 6, 7 AND 8

An engineer, who designs a bridge, a building framework, or even so simple a structure as a crane, must know in advance the stresses to which the various members of the structure will be subjected in order to specify the size of each part. The experience in the solution of such problems as is afforded by the following experiments is of value to everyone because of the development of a certain amount of common sense in problems of statics which serves as a very good guide in the absence of an exact solution.

Before attempting the experiments the following section on Statics should be studied. This section serves simply as a brief review of the conditions of equilibrium of forces and not as a complete text on the subject of Statics.

STATICS

GENERAL EQUILIBRIUM CONDITIONS FOR COPLANAR FORCES

Law 1. *The sum of the components in any direction of all the forces acting on a body equals the sum of the components in the opposite direction of the same forces.* This states that the body will not move in the particular direction chosen. It is usual to choose two directions of resolution at right angles to each other, and often these orthogonal directions are the vertical and horizontal directions.

An equivalent statement of Law 1 is as follows: All forces acting at a point, or on a body, or on any group of bodies which is in equilibrium, form a closed polygon if drawn end to end and all pointing around the polygon in the same direction.

Law 2. *The sum of all of the positive moments or torques about any point of all the forces acting upon a body or group of bodies in equilibrium, is equal to the sum of all of the negative moments or torques of the forces about the same point.* Positive moments or torques and the negative moments or torques tend to turn in the opposite direction about the point. This law states that there is no tendency of the body or group of bodies as a whole to turn about any point.

5. CRANE

Object. This experiment is intended to give practice in the solution of a few problems in statics dealing with the simple crane. The immediate object of the work is to determine, partly by graphical methods and partly by calculation, the forces acting to support the jib of a small crane in several arrangements of the apparatus, and to check these results with the measurement of the forces as given by spring balances.

Apparatus. The *small* crane, as shown in Figs. 10–13, consists of a jib having upon one end a pulley for accommodating either the supporting rope, or one carrying the weight or load. A steel pin, which passes transversely through the other end of the jib, passes loosely through two large holes or slots, one on each side of the jib, in the iron supporting frame. A vertical row of large hooks on the board back of the jib serves as convenient points to which the ropes can be attached. The centers of the circular hooks are in a vertical line passing through the pin at the lower end of the jib.

Procedure. Four separate cases are indicated in Figs. 10–13. The case indicated by Fig. 10 is the simplest possible; each succeeding case becomes more and more complex. It is, therefore, necessary that each one be perfectly understood before proceeding to the next. Each student is to do three of the four cases indicated below.

Each case is to be solved so far as possible by the graphical method. In this method of solution, an accurate drawing to scale is made of the structure under test. All dimensions used in the solution are then obtained directly from the drawing. All forces are represented in direction and magnitude by lines, and all resolutions into components and compositions of forces are made directly on the drawing. This method of attack, besides being much easier than the purely analytical method, is very much more instructive.

All dimensions necessary to make the accurate scale drawing of each set-up are to be taken in the laboratory. Each drawing should be accurately and neatly executed, and of sufficient size to give accuracy in the measurement of the distances. Each measured force is to be represented in magnitude and direction by a line drawn to some convenient and sufficiently large scale, and its magnitude expressed in pounds along the line representing the force. Forces are to be measured to tenths of a pound. The forces obtained *by calculation* are to be represented on the drawing in the same manner as indicated above for the *measured* forces, and the numbers representing their magnitudes printed in a different colored ink or bracketed to distinguish them from the measured values. If the direction of a calculated force does not coincide with

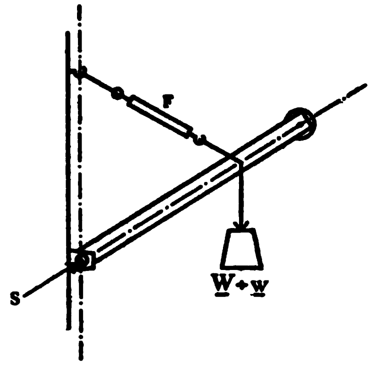

FIG. 9

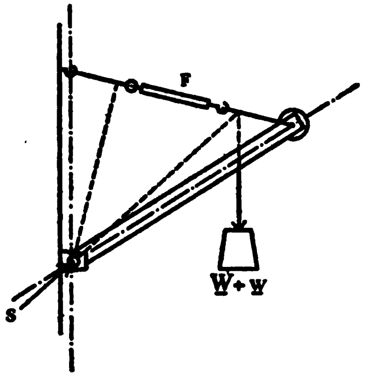

FIG. 10

the direction of the same force as measured, the calculated force should be clearly represented in some way, as by a different colored ink, or by a dash line.

Referring to Figs. 9–12, *W* represents the weight hung on as load and should be from 8 to 12 lbs. The weight of the jib, denoted by *w*, acts as if concentrated at its center of gravity. *F* and *S* are supporting forces in the tie rope and at the base of the jib, respectively.

In Case 1, Fig. 9, an additional weight *W* is hung at the center of gravity of the jib. This case is exactly equivalent to a weightless beam and a weight $W + w$ supported by forces *F* and *S*. Obviously, in this particular case, the force *S* is in the direction of the axis of the jib.

The force *F* and the vertical and horizontal components of *S* are to be measured in the laboratory. In a practical case only those forces which are underlined would be known, and the others would have to be calculated. Consequently, in working out the results, the underlined forces are to be considered as known, and the other forces are to be obtained by calculation and compared with the measured values. All dimensions necessary for an accurate drawing are to be taken.

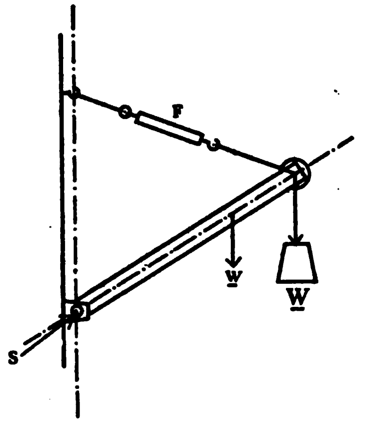

Fig. 11

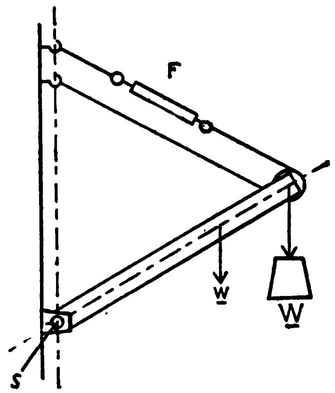

Fig. 12

The following is a detailed list of the measurements necessary for Case 1.

Case 1.

1. Weigh the load to determine *W* to tenths of a pound.

2. Record the exact position and magnitude of w, and important dimensions of the jib.
3. Arrange the apparatus as shown in Fig. 9, and record F to nearest tenth of a pound.
4. Measure the exact distance from the center of the pin at the lower end of jib to the center of the small circle of the hook to which F is attached.

   NOTE. The string attached to the hook always has a direction which passes through the center of the hook circle.

5. Two spring balances are to be fastened with ropes to the pin in the lower end of the jib. One balance must be pulled vertically upward until the pin just clears the lower edges of its slots in the supporting frame. The second balance is then pulled in a horizontal direction until the pressure of the pin against the backs of the slots is just overcome. Simultaneous readings of the balances give the vertical and horizontal components from which S may be found. Show that S in Case 1 is in the direction of the jib.

The arrangement described above is a very special case and not often found in practice. The other cases are not so simple, for, in general, the forces are not applied at the center of gravity, and the weight of the beam causes the supporting force S to act in some direction other than the direction of the axis of the jib.

Case 2, shown in Fig. 10, represents an arrangement one step more difficult than Case 1. All dimensions and forces are to be measured as in Case 1. Cases 3 and 4 are indicated in Figs 11 and 12, respectively.

Results. The following specific directions are given for Cases 1 and 2, but the other case is to be worked out without further aid.

Case 1.

1. Draw the complete and accurate drawing to scale, making the drawing as large as convenient on a sheet of report paper, and letter the drawing.
2. Record the magnitudes of measured forces as described.
3. Lay off the magnitude of the force $W + w$ to some proper and convenient scale.

4. In this special case, use the polygon-of-force method, and complete the parallelogram. Determine, from the scale, the values of F and S, and record these numbers beside the lines representing the respective forces. Record also the measured values of F and of the components of S.

Case 2.

1. The drawing should be made as for Case 1.
2. In this case, since the magnitude of F and both the magnitude and direction of S are unknown, the method of the first case cannot be used. The problem must be solved by moments. Draw the dotted line perpendicular to the direction of F. This is the moment arm of the force F about the pin as an axis.
3. Using Law 2, express the moments of $W + w$ and F about the pin as an axis. F, being the only unknown force, can be calculated.
4. Lay off to the proper scale the magnitude of F, and resolve F graphically into its horizontal and vertical components.
5. By means of Law 2, the horizontal and vertical components of S can be calculated and should be compared with the observed components.
6. From each set of components, the magnitude and direction of S can be obtained and expressed according to the directions.

Work out one of the cases completely before leaving the laboratory.

6. ROOF-TRUSS

Read *Introduction to Experiments 5, 6, 7, and 8.*

Object. The immediate object of this experiment is to obtain, by experiment and by calculation, the forces acting upon the roof structure as a whole and upon its parts under different conditions of loading.

Apparatus. A roof-truss, provided with weights and spring balances, is provided for use in this experiment.

The *roof-truss,* shown in Fig. 13, consists of two members hinged together at their upper ends corresponding to the position of the ridge-pole. The lower end of one beam is hinged to a support. The lower end of the other beam rests freely on a block of wood, and is constrained horizontally by a rope extending between the lower ends of the two beams.

The truss as a whole is acted upon by the external forces V_1, V_2, W, w_1, and w_2. The left-hand member is acted upon by the forces V_1, F_1, w_1 and R_1, the latter force being the force which acts *upon* the upper end of the left-hand beam. This force R_1 must cause an equal and opposite reaction, which force is the stress in the end of the left-hand member. This whole discussion, which applies to the left-hand members, applies in principle to the other member.

In any actual case, only those forces underlined in the figure would be known. Consequently, in the solution of the problem as given in this experiment, the underlined forces are assumed known, and the forces not underlined are to be calculated and compared with the measured values as obtained in the laboratory.

Procedure. Three cases will be solved. Case 1 represents a truss loaded at the central point corresponding to the ridge-pole. Case 2 represents a roof-truss loaded on one side with snow. Case 3 is to be an original arrangement and is to be different from the two cases given above.

The specific directions are given below for Case 1.

Case 1. Load down the roof-truss by a weight of about 10 or 15 lbs. hung at the center. Pull vertically upward on the end of the member which rests upon the block, and then carefully let it down, always keeping the balance and supporting rope vertical. This insures that the force between the truss and the block is vertical.

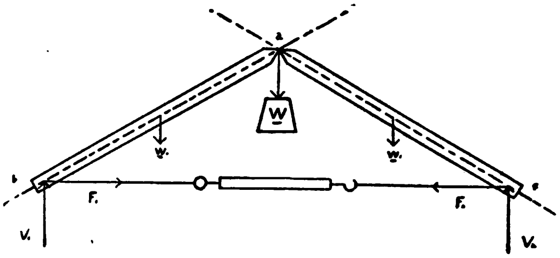

FIG. 13

Measure carefully all dimensions necessary for an accurate drawing of the truss as set up. Read the balance in the tie rope and, by means of another balance, the force necessary to just equalize V_2. In reading these balances, it is well to support the balance in the tie rope. Weigh the weight W with a spring balance to 1/10 pound. The forces R_1 and R_2 cannot easily be measured in the laboratory, but will be calculated. The weight of each member w_1 and w_2, which may be assumed the same, can be obtained by lifting the unloaded truss from one end, thus measuring the upward force corresponding to V_1 or V_2, which are in this case equal.

Case 2. Fasten along one of the beams a bar of metal to represent a snow load on one side of the roof. Omit W. Measure all dimensions and forces as in the preceding case. To find V_1 and V_2, lift the truss from the free end, shifting the load of "snow" from one side to the other.

Case 3. The third case is to be an original set-up as different from Cases 1 and 2 as possible. A very interesting case would be an arrangement in which one end of the truss is at a different level than the other end. The complete solution is to be made as before.

Computation and Results. The solution for obtaining the values of the forces not underlined in the figure is to be made according to the outline given below. All necessary distances to be used in the moment equations are to be obtained from the accurate drawings to be made for each case. It is clear, that since the members have weight, the resultant of the F and V forces, and the R force cannot be assumed in the direction of the beam. The solution cannot, therefore, be made by the usual parallelogram-of-forces method, but must be made by the combined application of Laws 1 and 2 in the section on *Statics*.

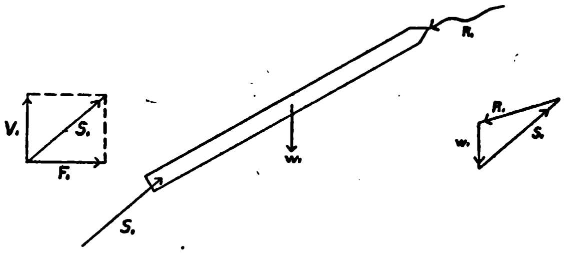

FIG. 14

The solution of problems by moments is much easier if an axis be chosen about which the moment of at least one of the unknown forces will be zero. A little experience helps in the choice of the shortest methods. An outline of a method for the solution of Case 1 is given below.

1. To determine V_2, consider the structure *as a whole*, and take moments about point *b* of only those forces which act upon the truss *as a whole*.
2. To determine V_1, express the moments about point *c*, or use Law 1 for the forces acting upon the truss as a whole.
3. To determine F_1, take moments about point *a* of all forces acting upon the left-hand member. Law 2. Necessarily, $F_1 = F_2$ in magnitude.
4. Obtain S_1, the resultant of V_1 and F_1, by the parallelogram law. A possible resolution is shown in the left-hand side of Fig. 14. The same method gives S_2.
5. R_1 is obtained as follows: make an isolation sketch, such as the one in the middle of Fig. 14, for the left-hand member of the truss. The forces S_1 and w_1 act as indicated, while R_1 is drawn as a wavy line showing that its direction is unknown. R_1 is now found by means of a force polygon. To do this, vectors representing the known forces S_1 and w_1 to any convenient scale are drawn end to end, thus forming all but one side of a polygon. This remaining side represents in magnitude and direction the desired force R_1, as shown in the right-hand sketch in Fig. 14.
6. Find R_2 by the method of 5.
7. Make an isolation sketch for the three forces acting on the pin at a. Draw a force polygon for them as a check on your previous results.

Cases 2 and 3 are to be solved in a similar manner and without further aid. The results are to be expressed in the customary way.

7. LARGE CRANE

Read *Introduction to Experiments 5, 6, 7, and 8.*

Object. The immediate object of the following experiment is to obtain all forces acting in a large model of a loaded crane, and thereby to give a practical knowledge of the stresses to be expected in an actual crane, and the method of finding the exact value of the forces.

Apparatus. The model of a crane to be used in this experiment is shown in Fig. 15. The main vertical beam rests on a platform balance which gives the total vertical force V_2. The upper end of the beam is guyed in the direction of H_1 by a rope and spring balance, the balance serving to give the value of H_1. An iron link supports the top of the beam in a direction perpendicular to the plane of the paper, but is constructed so that it does not affect the forces in this plane. A steel pin passes horizontally through the lower end of the jib and works freely in slots in the iron side pieces at the middle of the vertical beam. The lower end of the jib is supported vertically by two spring balances, one connected to each end of the pin. The four pulleys, two at the upper end of each member, are for accommodating the rope, and may be used singly as shown in the figure, or together to form double pulley blocks.

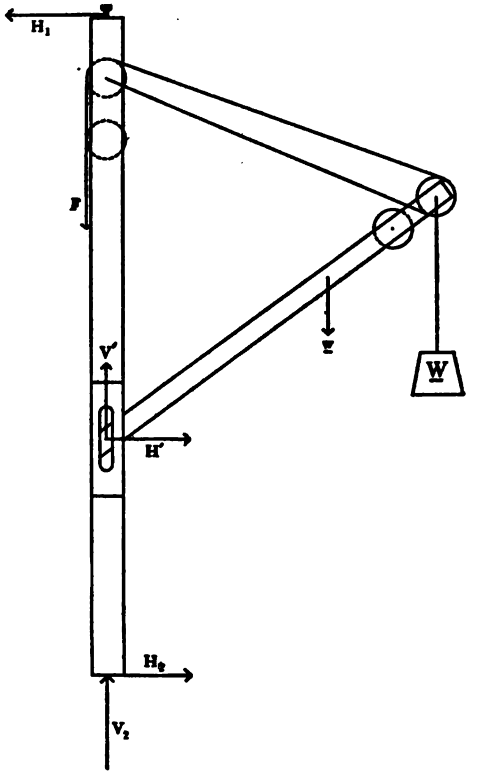

Fig. 15

Procedure. Arrange the crane as shown in the diagram, and load with a weight of over 50 pounds. Measure all dimensions necessary for making an accurate drawing to scale of the crane as arranged. Measure all forces shown in the diagram. The weights of the beams are given on the apparatus.

The force H' is determined by pulling horizontally with a spring balance until the pressure of the pin against the side of the slot through which it works is relieved. The two balances which give the value of V' should be read only while force H' is being determined, for otherwise the friction between the pin through the jib and the side of the iron slot would cause a large error.

The lower end of the vertical beam rests on balls which are free to roll in races in a stationary iron plate. The force H_2 can, therefore, be measured, as was H', by pulling horizontally with a spring balance until the lower end of the crane just moves. The reading of V_2 and H_2 should be taken simultaneously, for otherwise the side thrust of the lower end of the crane would make the reading of the platform balance indefinite and inaccurate.

A second original case is to be set up, and the data recorded.

Computation. The forces which are underlined in the figure are those which would be known in any actual problem, and are, consequently, the only forces to be assumed as known in the solution of the two cases to be solved. All forces not underlined are to be obtained by methods similar to those outlined for the small crane and roof-truss. The final resultants of all component forces are to be indicated on the drawings.

8. THE BRIDGE TRUSS

Object. To study the forces in a loaded bridge truss; to discover which members are in tension, and which in compression; to observe the amount of the force in each member, and to compare these with the amounts given by calculation.

Apparatus. Experimental Bridge Truss, with test member and spring balances; heavy weight, as a load.

Method. Hang the heavy weight on the center bar of the bridge. This load produces stresses in each member of the bridge which may be measured by means of the special test member. The latter is to be attached in such a way that the force in it will overcome the tension or compression in the bridge member, and leave the latter slack. The sum of the readings in the two spring balances of the test member will then give the required stress. The force in the test member can be adjusted by means of turnbuckles, and can be made either a tension or a compression by reversing one end. A little study of the apparatus will make this plain.

If the load were lifted off the bridge, there would still be forces in the bridge members due to the weight of the bridge itself. These are to be found by trial, after the loaded bridge is finished. The difference between the observed forces in a given member when loaded and when not loaded gives the force produced by the load itself, and this is the quantity which is to be compared with the calculated value.

In order to calculate the force produced in each member by the load, let us first consider one end of the bridge. Fig. 16 represents the whole bridge. R_1 and R_2 will, by symmetry, each equal half the load in this case. Fig. 17 is what is called an *isolation sketch* of the pin at A; that is, it is an enlarged view of the pin "isolated" or separated from its surroundings, with all the forces that act on it indicated by arrows. These are the thrust of the supports $\left(\frac{W}{2}\right)$, the thrust F_1 of the diagonal member AC down and to the left, and the pull F_2 of the horizontal member AB to the right. We know by inspection that F_1 must point as shown, and not in the opposite direction, because the sum of all the forces or components of forces upward must equal the sum of all the forces or components of forces downward if the pin is to be in equilibrium. Now $\frac{W}{2}$ points up, and F_2 does not point either up or down; so F_1 must point in such a way as to have a downward component to balance $\frac{W}{2}$. Similarly for right and left equilibrium, F_2 must point

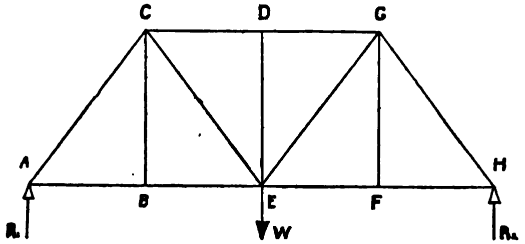

FIG. 16

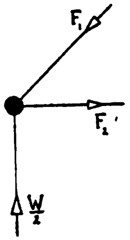

FIG. 17

toward the right to balance the leftward component of F_1. That is, the member AC is in compression and the member AB in tension.

We now solve for F_1 and F_2, either by the parallelogram of forces, or by the *triangle of forces* (which amounts to the same thing). If the sides of ABC are 1.5 feet, 2 feet and 2.5 feet, we have, since ABC is similar to the force triangle

$$\frac{F_2}{3/2} = \frac{\frac{W}{2}}{2} = \frac{F_1}{5/2}$$

or $$F_1 = \frac{5}{2} \times \frac{W}{4} = \frac{5W}{8}$$

and $$F_2 = \frac{3}{2} \times \frac{W}{4} = \frac{3W}{8}$$

In a similar manner, consider the point B, and work out the forces in BE and in BC. Then, by considering the horizontal and the vertical forces at the point C, find the stress in CD and in CE. Finally, by considering the forces at D, find the force in DE. By symmetry, the forces in the other half of the bridge may be written in without further calculation.

If there is time, repeat the whole experiment, starting with the load on the pin B. In this case, there is no symmetry about the arrangement, but the calculations can be carried through in much the same manner as before. The calculations for this case are to be made, even if there was not time to make the corresponding measurements.

Present the results in the form of a bridge diagram, marking tensions or compressions by arrows suitably directed, and writing the forces, both calculated and observed, beside the corresponding bridge members.

9. CENTRIPETAL FORCE

Object. The object of this experiment is to study and test the laws of centripetal force.

Apparatus. The apparatus consists of a disc, capable of revolving, with some small brass stops fastened to it. Some small rectangular steel blocks, a tuning fork, and smoked paper in strips are also provided.

Theory. If a block is set up on the disc with one edge resting against one of the brass stops, and the disc is then made to revolve faster and faster, a speed will be reached at which the block will tip over. Let h and b denote the height and breadth of the block, and let a_t denote the acceleration of the block when tipping occurs. It may be proved that

$$a_t = gb/h \quad \ldots\ldots\ldots\ldots (1)$$

But the acceleration a_t is also given by the general formula

$$a_t = v^2{}_t/r \quad \ldots\ldots\ldots\ldots (2)$$

where v_t is the velocity of the block when tipping occurs and r is the distance of its center of gravity from the axis of rotation. In this experiment we compare the values of the acceleration given by the right-hand members of (1) and (2) and thus check the two formulae. For this purpose we measure the values of v_t corresponding to various values of b, h, and r,

Proof of (1): Consider the forces acting on the block before it reaches a speed sufficient to produce tipping. The block has an acceleration a directed towards the axis of the table. Hence the resultant force acting on it must be $\frac{W}{g}\,a$, or Ma, and must be directed toward the axis of the table. Furthermore, so long as the block does not tip it is in equilibrium as regards rotation about its own center of gravity. This means that the sum of the moments of the applied forces about the center of gravity must be zero.

The applied forces consist of (1) the force of gravity W acting downward through the center of gravity, (2) an equal and opposite force R due to the reaction of the table, and (3) a horizontal force F acting along the line of the base of the block and due to the pressure of the block against the stop. The resultant of these three forces is F and consequently $F = Wa/g$. The moment of F about the center of gravity is $Fh/2$ or $\frac{Wah}{2g}$. The moment of W is zero and hence the moment of F must be balanced by an equal moment due to R. Then obviously R cannot act through the center of gravity. Let d denote the distance from the center of the block to the line of action of R. Then if the block is not to tip, d must satisfy the equation

$$Rd = Wd = \frac{Wah}{2g}$$

$$d = ah/2g$$

As the speed of rotation increases the force F and the corresponding moment increase. Hence d must increase and the point of application of R must shift outward to preserve equilibrium. This continues until finally d becomes equal to $b/2$ and the force R acts up through the outer edge of the block. If the acceleration increases beyond the corresponding critical value equilibrium cannot be maintained and the block will tip over. Hence

$$b/2 = a_t h/2g$$

which leads immediately to equation (1).

Procedure. See that the disc is accurately level. For each test, place the block against the appropriate stop, then slowly speed up the disc by turning the axis. When the block tips over, stop accelerating the disc, and take a tuning fork record *immediately*, to get the value of v_t. The tuning fork record is obtained by pressing the vibrating fork, fitted with a fine tracing wire, or bristle, gently against a strip of smoked paper, placed around the edge of the disc. A fresh strip is to be placed in position for each test.

Each set of apparatus includes 3 blocks with square ends and lengths equal to 2, 3, and 4 times the width of the ends. By standing them on end, therefore, one may make b/h take the values 1/4, 1/3, or 1/2; by laying any one, preferably the shortest, on its side, pointing in a direction across the radius, one obtains $b/h = 1/1$. By laying them on their sides along the radius, we have $b/h = 2/1$, 3/1, or 4/1. The nearer edges of the brass stops are at such distances from the axis as to make the centers of gravity of the blocks, when properly placed against them, just 10 cm. from the axis. Each stop is, of course, good for any block with a certain value of b, so that the same one will do for any value of b/h up to 1/1, and others must be used for $b/h = 2/1$, or 3/1. Each stop is marked with the values of b/h for which it is intended.

Record the vibration number of the fork.

In carrying out the experiment, make tests at the 10 cm. radius for values of $b/h = 1/4, 1/3, 1/2, 1/1, 2/1$, and 3/1.

Results. The results are certain values of a_t. One set is obtained from the measurements, the other from the formula involving the dimensions of the blocks. Plot two curves on the same sheet of paper, with values of v_t as abscissae for both curves. For one of these, use $a_t = g\frac{b}{h}$ as ordinates, placing each value opposite the corresponding observed value of v_t. For the other curve, take any convenient values of v_t, calculate $\frac{v^2_t}{r}$ from these, and use these values of a_t as ordinates; this curve will be the theoretical curve.

What form of graph should we have if we plotted a_t against r?

10. STUDY OF A SPRING

Object. To load a spring with weights, and to determine how the extension varies with the force; also to find how the period of vibration of the spring varies with different loads.

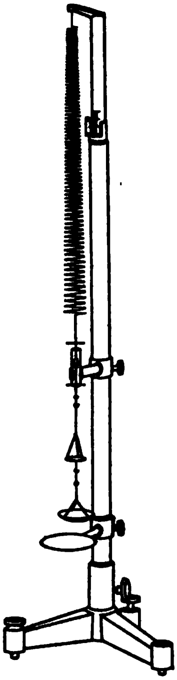

Fig. 18

Apparatus. Spiral spring mounted in a Joly balance; weights; stop-clock.

The Joly Balance

The Joly balance, shown in Fig. 18, is essentially a very sensitive spring balance. The top end of the spring is attached to a support which is movable up and down by means of an easy and convenient control located at the bottom of the upright. The distance the top is raised is indicated by a scale and vernier on the vertical standard of the instrument. The manipulation of the instrument consists in placing in the pan, which is attached to the bottom end of the spring, the object to be weighed, and raising, by means of the adjusting mechanism, the top of the spring until a line on a piece of metal hung on the lower end returns to a certain stationary index mark.

The stationary index mark is a scratch around the whole circumference but on the inside of a piece of glass tubing clamped on the main support and through which the piece of metal, hung on the lower end of the spring, freely passes. In viewing the index mark, the experimenter must be in such a position that the scratch on the glass toward the observer just hides the scratch on the back half of the circumference. The setting consists in bringing in line with this stationary index scratch the middle white line on the movable piece of metal.

Method. *First part.* Level and adjust the balance so that the parts are in line, and the spring system passes centrally through the glass tube upon which the reference mark is scratched. Two scale pans are hung on the spring; one of these is to dip into a glass of water, so that it hangs quite free, and only a single wire protrudes through the surface of the water. In this experiment this lower pan is used merely to damp out the vibrations of the spring, and so to make possible much quicker readings of the balance.

Take the zero reading, with no weights on the pans. This should be not far from zero, but no attempt should be made to make it exactly zero. Make several independent settings, and average. Then place a weight in the upper pan and obtain the reading of the balance for this weight by at least two trials. Repeat this process for several other weights, progressing up to the limit of the balance by steps sufficient to increase the stretch of the spring each time by about 5 cms.

In the report on this part of the experiment, a curve is to be plotted for which the abscissae represent the loads, and the ordinates the elongations. Observed points are to be represented by dots in small circles, and a smooth line is to be drawn, not necessarily through each point, but following the general course of all the points. If this line is a straight line, it means that the spring follows Hooke's law, $F = kx$, where x is the elongation in centimeters produced by a force of F dynes, and k may be called the stiffness of the spring.

Second Part. If the spring is stretched a little and then let go, it will vibrate with simple harmonic motion. As explained in the text-books (*e. g.* Kimball's College Physics, page 84) the time of a complete vibration is $2\pi\sqrt{\frac{m}{k}}$ where m is the mass moving, expressed in grams, and k is the "stiffness." The mass here referred to is the mass which would have to be hung on an ideal weightless spring of the same stiffness as the actual one, in order to produce the same rate of vibration.

Measure carefully, by at least two trials of 50 or 100 swings each, of small amplitude, the rate of vibration of the spring, removing for this purpose the glass of water, and the lower pan, and attaching the upper pan directly to the spring, the index device being unhooked and pushed aside. Hence calculate the equivalent mass of the spring (and pan). Should this be the same as the actual mass ? Test it, if in doubt, by weighing the spring and pan. Explain.

As a test of the relation between time, mass and stiffness, measure the time of swing, as before with, say, 5 grams on the pan, and again with 10 (or choose 3 and 6 if the spring will not support 10). In each case calculate what ought to be the period of vibration with the total mass (equivalent mass of spring plus load), and compare it with the time actually observed.

Estimate, with reasons given, the percentage accuracy with which the time of vibration can be measured, and the error which is introduced, from this cause, in the comparison between theory and experiment. How could this error be reduced ?

11. THE PENDULUM

Object. The object of this experiment is to make an accurate measurement of the value of g.

Apparatus. Bifilar pendulum; clock; cathetometer.

Method. According to the " pendulum formula," the value of g can be found by measuring the length of a simple pendulum, and its time of swing. The formula is true only for a simple pendulum, consisting of a particle on the end of a weightless string, oscillating through a very small arc. In practice, the particle must be heavy, if it is to swing long enough to enable us to measure its period accurately, and in that case the simple formula no longer holds precisely; and the length of the pendulum is not so easy to measure.

Better precision is obtained if the bob is in the form of a cylinder, supported by parallel and equal wires in a bifilar arrangement. In this case the bob remains parallel to itself, and each particle of it moves in a different circle about a different center, and therefore acts like a simple pendulum. The radii of these circles are all equal, and equal to the length of the wires.

In this apparatus, the wires are double, and are looped over small pins, which project at the ends of the bob, and the supporting bracket. The wire will rock in its place, and its "length" will be from the upper part of the upper pin to the lower part of the lower. This distance can be measured without touching the pendulum, by means of the cathetometer. This instrument is a vertical scale with telescope and vernier. There is a sliding piece which can be clamped at any height, and which carries a screw by means of which the telescope, when clamped, may be raised or lowered through small distances. This is the fine-adjustment screw. The telescope has cross-hairs, the intersection of which is to be set level with the upper point; and the vernier then gives the upper reading. By loosening the clamp, and letting down the telescope to such a place as permits the lower mark to be seen, and again clamping and using the fine adjustment screw until the cross-hair passes through the lower mark, a second vernier reading is obtained. The difference of these two readings is, of course, the length of the pendulum. This is to be found for *each* wire, preferably measuring the two upper marks first, then dropping down to the two lower. Average the two lengths.

The cathetometer scale is of brass, and is correct only at a certain temperature marked on a tag on the instrument. A thermometer nearby gives its temperature. Assume an expansion coefficient of 0.000018 for brass, and correct the lengths as measured for the expansion of the scale.

To obtain the time of swing, set the pendulum swinging through a small arc only, say one or two cm. each way from the position of rest, and be sure that it moves parallel to the wall, without any wobbling. When this condition is secured, close the light switch on the shelf, and watch the behavior of the small lamp. Current to light this lamp flows through the relay, run by the clock, and through the pendulum wires, leaving the bob by the fine wire projecting below, which cuts the mercury drop in each oscillation. The mercury drop will not, in general, be cut at the same instant as the relay is closed, so that, most of the time, no current will flow. Once every few minutes, however, a coincidence occurs between the motions of the clock and the pendulum, and when this happens the lamp will light up at the middle of each swing for several swings. During the interval between two such coincidences the pendulum will have gained, or lost, one swing, or half period, over the clock. Determine which. If we note the time between coincidences, in seconds, we have the number of clock swings (half periods) in this interval, which is the same as the number of seconds; and the number of swings of the pendulum is one more, or less. Hence we can find the time of one swing, and the period of the pendulum.

In practice, note the time on the clock face (in hours, minutes and seconds) when the first and last flashes of each coincidence occur, and take their mean as the true time of the coincidence. Measure, without stopping the pendulum, the interval of time between at least five consecutive coincidence periods, for greater accuracy.

Discuss the sources of error in the experiment, and estimate the percentage error that might reasonably be made by this method, both in the time and in the length. Find the likely percentage error in your value of g.

INTRODUCTION TO EXPERIMENTS 12, 13, AND 14

Experiments 12, 13, and 14 are alike in dealing with the capacity of a machine to do that for which it is designed. The efficiency of a machine, whether it be a steam engine, an electric motor, a jack screw, or what not, is of great practical interest because it expresses the ability of the device to transform into useful effects the work which is supplied.

The experiments are of value in giving an opportunity for observing the operation and principle of two forms of machines, and in illustrating some common engineering methods of measuring efficiency. The results are very interesting in showing the comparatively low efficiency of some common devices.

EFFICIENCY

The *efficiency* of a machine is the ratio of the power output to the simultaneous power input, or the efficiency may be expressed as the output of the machine in useful work for a certain interval of time, divided by the work input for the same interval of time, the two being expressed in the same units. Efficiency is usually expressed in per cent.

If the machine is one which does not run continuously as, for instance, a lever, or a chain hoist, it is necessary, in order to find the efficiency, to do a measured amount of work upon the machine, and measure the amount of useful work which results.

If the machine runs continuously, it is customary to measure the amount of work put in for some interval of time, say a second or a minute, and *at the same time* measure the amount of useful work performed by the machine for the same interval of time. In some special cases it may be easier to measure the input or the output, and to measure or to calculate the losses, from which data the efficiency can be determined. This is often the case with electrical machines.

For practically every machine the efficiency depends upon the conditions of load. In general, the efficiency increases up to a certain maximum as the machine is caused to do more and more work, and then finally drops off as the machine is loaded down still more. The load for which the machine gives its maximum efficiency is the load for which the machine is designed.

It is, therefore, usually of great interest to have the complete history of the way in which the efficiency changes with increasing load. This history is usually plotted in an *efficiency curve,* for which the ordinates represent the efficiencies corresponding to the outputs as abscissae. When the machine is operating with no load or output, the efficiency is obviously zero. Both scales should, therefore, begin with zero at the origin. The efficiency curve is always a smooth curve.

The shape of the efficiency curve is of practical interest in showing the maximum efficiency attainable by the machine, for what load the machine works at this maximum efficiency, how rapidly the efficiency falls off as the load is either increased or decreased from the load for maximum efficiency, etc.

The following section on *Torque and Measurement of Work and Power* will be found helpful in working out the experiments on efficiency, and should be studied in preparation for these experiments.

The section on *Mechanical Advantage of a Machine* must be studied in preparation for Exp. 12.

TORQUE AND THE MEASUREMENT OF WORK AND POWER

The tendency of a force to produce rotation about an axis is dependent not only upon the magnitude of the force, but upon the *perpendicular* distance from the axis to the line of action of the force. The product of the force and this perpendicular distance, known as the *torque,* or *moment-arm,* measures the tendency of the force to cause rotation, and is called the *torque* or *moment of the force* about the axis. If the force is in pounds and the distance in feet, the torque is expressed as so many *pound-feet,* but not *foot-pounds,* which is the unit of work. A torque is the product of a force and a distance measured *perpendicular* to the line of action of the force, whereas work is the product of a force and a distance measured in the *same* direction as the force.

Torques or moments are useful in the solution of problems on equilibrium of forces as explained in the section on *Statics,* and also in the measurement of the work done by a machine provided with a brake. There are numerous other uses for the above principles which will appear later.

A force F which has a torque-arm r has a torque T about the axis, where

$$T = F \times r.$$

The force F is distant r from the axis, and, if allowed to cause the rotation, moves in a circle of radius r and circumference $2\pi r$. The work done in one revolution is $2\pi rF$ or $2\pi T$. In any case, where the torque causes constant rotation, the power developed is readily calculated if the speed of rotation is known. If n is the number of revolutions per second, the work done per second or the rate of doing work, which is the power, is equal to $2\pi Tn$. If the force and distance are expressed in pounds and feet respectively, the power in horse-power is $\frac{2\pi nT}{550}$.

Brakes are often applied to machines to measure the work done in a given time or the power output. Practically all brakes depend upon friction between rubbing surfaces, the work being dissipated in heat. If the amount of power is large, the brake must be cooled by water or by other means. Brakes are of various kinds, but all are provided with an arrangement by means of which the torque can be measured. The power can thus be calculated from the torque and speed.

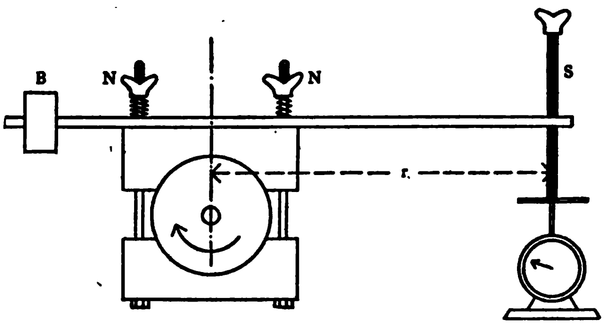

FIG. 19

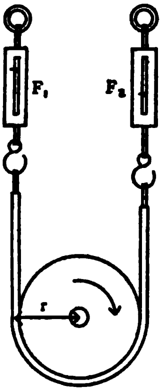

FIG. 20

Figs. 19 and 20 represent two common forms of brakes, each of which may be considerably varied in form. The brake shown in Fig. 19 is known as a *Prony* brake.

Mechanical Advantage of a Machine

When a mechanical force f acting upon a machine results in a force F exerted by the machine, as, for instance, in the case of the lever, the jack screw, etc., the ratio F/f is called the *actual mechanical advantage* of the machine. This mechanical advantage depends upon the amount of friction, and is, therefore, a variable according to the conditions under which the machine is operated.

The *theoretical mechanical advantage* of a machine is the ratio of the corresponding forces assuming *no* friction or 100% efficiency. The theoretical mechanical advantage is, therefore, dependent only upon the kind and dimensions of the machine, and is a constant. It is obtained from the assumption that the output equals the input. Let d be the distance through which the force f acting upon the machine moves in a particular case, and D the corresponding distance moved by the force exerted by the machine. If F' is the force that the machine ought to exert if there were no friction, then F' is greater than F. Also $f \times d = F' \times D$ if there is no friction. The theoretical mechanical advantage is, therefore

$$T.M.A. = \frac{F'}{f} = \frac{d}{D}.$$

F' cannot be measured, but the distances d and D can, and supply the data for calculating the $T.M.A.$

12. EFFICIENCY OF A CHAIN HOIST

Read the *Introduction to Experiments 12, 13, and 14.*

Object. The object of the following experiment is to obtain the efficiency curve and the theoretical mechanical advantage of two common forms of chain hoists.

Apparatus. A duplex chain hoist of one half ton capacity, a common differential chain hoist of one quarter ton capacity and a block and tackle are provided for test. Weights are provided for load, and spring balances will be used to indicate the pull on the chain necessary to operate the hoist.

The duplex pulley provided is a form of machine for the purpose of lifting heavy weights. It, in its results, closely resembles the common differential pulley hoist, but works on a slightly different principle. The important part of the mechanism is a worm and gear.

A description of the common differential pulley will be found in most text-books.

Procedure. Each student is to make measurements on two hoists. The theoretical mechanical advantage of each hoist is first to be determined. The hoist is to be operated, and the corresponding distances moved by the operating force and the resulting force are to be accurately measured. The distances moved must be large enough to give sufficient accuracy in the measurements. It is suggested that the output end of the machine is made to move at least a foot. The two distances can be measured in any units, provided the units are the same for both, for the distances always enter as a ratio, but centimeters will be found more convenient for measurement.

The data for the efficiency curve are obtained as follows: Load the machine down by steps to a little more than that for which the machine is rated, determining in each case the force necessary to operate the hoist. Since the efficiency changes most rapidly for small loads, the loads should begin at zero and progress by very much smaller steps at first. In all, there should be at least 8 or 10 points on the efficiency curve.

Since the static friction is greater than the friction when the parts are in motion, the reading of the spring balance, which records the pull on the chain, should be taken when the chain is moving with constant velocity. A small spring balance is provided for getting this pull when the force is less than 30 pounds. A larger one is provided for the greater pulls.

Be sure that the total pull on the chain is recorded. Note that a correction is usually necessary, due to the weight of the balance itself, and sometimes a zero correction besides. The method of making these corrections should be clearly indicated in the report.

The observed force is that necessary to raise both hoist and load. It is this force which must be applied in every practical case, therefore no correction need be made to determine the force needed to raise the load alone.

Results. The results for each hoist are as follows:

1. Efficiency curve of chain hoist with load as abscissae.
2. Theoretical mechanical advantage of chain hoist.
3. Conclusions and remarks.

13. EFFICIENCY OF A WATER MOTOR

Read *Introduction to Experiments 12, 13, and 14.*

Object. The object of this experiment is to test a Pelton water motor, and to obtain curves of efficiency and speed under varying loads. Although the efficiency obtained for the small motor provided for test is lower than that of a larger motor, yet the results are typical in general characteristics, and the method is instructive.

Water Power and Water Motors

Water which is under pressure, or which is in an elevated position, possesses potential energy, and if allowed to escape or fall from its elevated reservoir can be made to do work. The amount of power available depends not only upon the pressure or height, but also upon the quantity of water discharged per unit of time. The same quantity of work could be done in a given time by a small amount of water discharged under high pressure, or falling from a great height, as might be done by a very large quantity of water discharging under low pressure, or falling through a small distance. The latter is sometimes known as a *low grade* water power, but may be just as useful as the high grade system. There are many examples of the low grade power in this part of the country, a very good example of which is the new power derived from the Mississippi river at Keokuk. The high grade powers are very common in Norway and Switzerland, where water is often drawn from heights of a thousand feet or more.

The two kinds of power require different types of water motors. The low grade power is converted by the so-called turbine wheels. The other type of motor which is applicable to the high heads and small amounts of water is known as the Pelton or jet motor, and is the form which will be tested in this experiment.

In the Pelton type of water motor, water issues in a powerful jet from a properly designed nozzle, and impinges on a series of buckets fastened on the periphery of the rotating wheel. Fig. 21 shows the general appearance of the rotor. In some motors there is more than one nozzle. The buckets are usually double and so arranged that the jet of water strikes the sharp-edged partition between the two halves of the bucket and is thereby deflected, one half the stream going into each pocket. Each half of the stream, in following the curved bottom of the bucket, is reversed in direction and leaves the bucket without interfering with the main jet. The maximum amount of work is derived from the jet of water when the bucket has one half the velocity of the issuing jet, for then the water leaves the bucket with zero velocity and, therefore, zero energy.

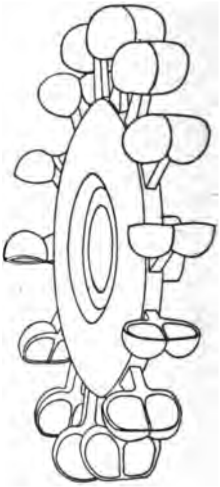

FIG. 21

Apparatus. The Pelton wheel, to be tested in this experiment, is operated at a pressure of about 90 pounds per square inch. This high pressure is supplied by a triplex pump operated by a small directcurrent electric motor. A relief valve is connected between the pump and the water motor and may be set at any pressure. The surplus water passes out through the valve and hence to the waste pipe. A pressure gauge indicates the pressure of the water. A water meter is connected in the supply to the water motor and may be used to determine the motor's consumption of water if only one motor is being operated, although in any case it is much more accurate to catch and weigh the quantity of water discharged by the motor in a given time. The motor is provided with a brake for measuring the power output of the machine as described in the *Section on Measurement of Torque and Power.*

A speed indicator is provided for determining the speed of the motor.

Procedure. An instructor will start the pump. The pressure should be kept constant, and readings of speed and torque are to be taken as the motor is loaded down from no load to a load sufficient to stop the motor. A preliminary run should be made in order to determine by what steps the motor should be loaded in order to give about 10 or 12 cases in the complete run. The quantity of water used remains about constant and may be determined only a few times. It will be necessary to know certain dimensions of the brake in the calculation of the power output.

Computation and Results. The power input of the water motor is to be calculated from the pressure of the water and the quantity of water which is used by the motor per unit of time. To calculate this input in foot-pounds per minute, multiply the pressure in pounds per square foot by the number of cubic feet of water flowing per minute.

The output of the motor is to be calculated as explained in the *Section on Measurement of Work and Power.*

The results are to be plotted in curves. There are various ways of doing this, the more usual being to plot curves of efficiency, speed, and torque as ordinates against power output as abscissae. In this case, however, curves of speed and efficiency are to be plotted as ordinates against torque as abscissae.

After plotting the curves, the following deductions from the curves are to be stated:

1. Maximum efficiency.
2. Speed at maximum efficiency.
3. Speed at no load.
4. Ratio of speed at maximum efficiency to speed at zero load.

14. EFFICIENCY OF A TRIPLEX PUMP

Read *Introduction to Experiments 12, 13, and 14.*

Object. The object of this experiment is to obtain a curve showing how the efficiency of the pump varies with the power output.

Apparatus. The pump provided for test is a commercial three-cylinder type known as a triplex pump. Fig. 22 gives, in diagram, the arrangement of the apparatus. The pump is driven through belt and gears by an electric motor *M*. Water is pumped from the barrel reservoir *R*, through the water meter *Q* and two pressure regulating valves, one of which is indicated at *V*. These relief valves are connected in series, and serve to regulate the pressure from zero to about 150 pounds per square inch against which the pump works. This back pressure is indicated by the gauge *P*.

Procedure. Simultaneous readings of pressure, quantity of water discharged per unit time, and input of the electric motor are to be taken for 8 or 10 adjustments ranging in about equal steps of pressure from zero to 120 pounds per square inch.

The quantity of water is best determined by noting the exact time taken by the one-cubic-foot hand of the water meter to make just one revolution. Take several observations of this time for each value of pressure.

The input to the electric motor is obtained from the readings of the ammeter I and the voltmeter E.

Computations and Results. To find the efficiency of the pump we must get its *input* and its *output.* The input (in foot-pounds per second) is found as follows: multiply each reading of the ammeter I (in amperes) by the corresponding reading of the voltmeter E (in volts); this gives the electrical input (in watts) to the motor. An efficiency curve of the motor is posted nearby from which the proper value of the efficiency is to be selected, and this is to be multiplied into the above product, thus giving the power output of the motor, or input to the pump. To convert this to foot-pounds per second multiply by $\frac{550}{746}$, or 0.737.

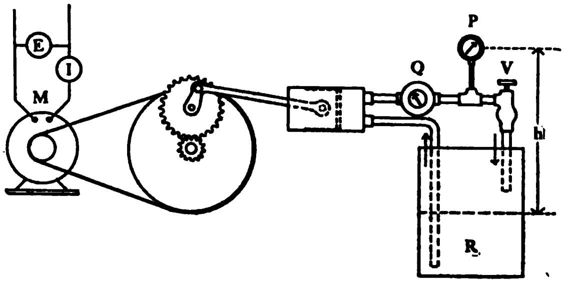

FIG. 22

The output is found as follows: pumping against a pressure of P pounds per square inch is equivalent to raising water to a height of $\frac{144}{62.5} \times P$ feet; pumping one cubic foot in t seconds is equivalent to pumping $\frac{62.5}{t}$ pounds in one second. Hence the product (height raised) × (weight lifted) = $144 \frac{P}{t}$ foot-pounds per second.

The results are to be expressed in a curve of efficiency (plotted as ordinates) against power output in foot-pounds per second (as abscissae).

15. MOMENT OF INERTIA

Introductory Remarks. The *mass* of a body is the important factor in determining the translational motion of the body in any direction when acted upon by a force in that direction. Similarly, the *moment of inertia* of a body is the property of the body which determines its rotational motion when acted upon by a force which has a moment about the axis of rotation.

The same analogy between mass and moment of inertia appears when expressing the energy of motion of a body. The kinetic energy of rotation of a body is expressed in terms of the moment of inertia of the body about the axis and its angular velocity of rotation, just as the kinetic energy of translation of a body is expressed in terms of its mass and linear velocity.

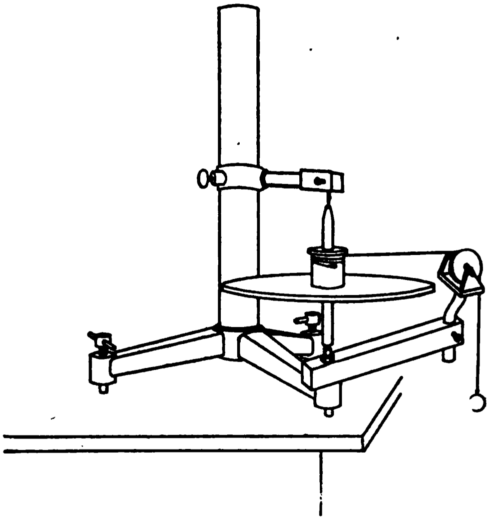

FIG. 23

There are many practical problems which deal directly with rotational motion, and, therefore, involve a moment of inertia. All problems dealing with rotating machinery fall in this class, and include such problems as the design and action of fly-wheels. There are many other problems such as the strength of beams as determined by the shape of their cross section which, although the reason may not be apparent, involve the moment of inertia of the cross section. A knowledge of the meaning of the moment of inertia and its use in simple problems is, therefore, of great importance.

Object. The following experiment is designed to illustrate the use of moment of inertia in the solution of problems in rotation. The moment of inertia of two bodies will be obtained by experiment, and the results for one of these compared with the value of the moment of inertia calculated from the mass and dimensions of this body.

Apparatus. The apparatus for this experiment, shown in Fig. 23, consists essentially of a vertical shaft carrying a large disk and a drum. The bearing at the lower end of the shaft consists of a highly polished, glass-hard, steel point which fits into a hard, polished cup on the end of the shaft. This bearing is made to have as little friction as possible, and because of its small size and the danger of injury, deserves special care. The bearing at the top consists of a steel point which fits into a hard steel cup in the upper end of the shaft. This bearing carries none of the vertical load, but serves to guide the upper end. A fine silk thread, to one end of which is attached a spherical weight, passes over a nearly frictionless pulley, and then horizontally to a drum on the shaft over which it winds. A ring, the moment of inertia of which is to be obtained, is provided for use with the apparatus.

Discussion. The ordinary formulae for accelerated motion in a straight line are

$$v = at$$
$$s = \tfrac{1}{2} a t^2$$
$$v = \sqrt{2as}$$

where s is the distance traversed from rest in the time t, when the motion is with acceleration a, and v is the velocity at time t.

Exactly similar formulae hold for rotational motion, and are as follows:

$$(1)\quad \omega = a t$$
$$(2)\quad \theta = \tfrac{1}{2} a t^2$$
$$(3)\quad \omega = \sqrt{2a\theta}$$

where ω is the angular velocity measured in radians per second,

t is the time, θ is the angle in radians swept out in time t, and α is the angular acceleration.

When a body of mass M is caused to move in a straight line by the application of a constant force F, the acceleration a of the body is given by the equation $F = Ma$.

If a body is caused to rotate through the action of a force, which has a moment about the axis, the angular acceleration is given by a formula exactly similar to the above, i. e.,

$$(4)\quad Fr = I\alpha$$

where Fr is the moment of the force F about the axis, I is the moment of inertia of the system about the axis, and α is the angular acceleration.

The kinetic energy of a mass moving in a straight line is

$$E = \tfrac{1}{2}Mv^2,$$

and the analogous expression for rotational kinetic energy is

$$(5)\quad E = \tfrac{1}{2}\,I\omega^2.$$

The moment of inertia of a rotating body may be found by the application of either formula (4) or (5). We shall make use of a method based on formula (4). If a weight, attached to a thread which winds over the drum on the shaft, starts from rest and descends through a measured vertical distance, the friction in the bearings and the air virtually reduces the pull on the drum, so that the resulting angular acceleration is not so large as would be expected; also the potential energy given up by the falling weight does not all reappear as kinetic energy of the disc, but some is lost on the way. These two facts are closely connected, and we shall use the latter to help in finding the reduction in force referred to in the former, and so in allowing for the effects of friction.

Procedure. Put a little oil on the bearings, and very carefully place the shaft and disk in place. The upper bearing should exert no downward pressure on the shaft. Using a spirit level, adjust the position of the upper bearing until the disk is exactly level.

Tie a weight to a light thread, the other end of which is tied into a hole at the edge of the drum; the thread being of such a length that the ball almost touches the floor when it is all unwound from the drum. Then turn the disc, winding up the thread on the drum, and raising the weight in all just 100 cm. *from its lowest position.* Then let it go. The disc will spin with increasing angular velocity as the weight falls, but, afterwards, the thread will wind itself up again, and the weight will rise to a height which would be the original height if it were not for friction. Measure the actual height reached above the lowest position of the weight. Call h_1 the height through which the weight falls and h_2 the height to which it subsequently rises. If F_1 is the force of friction, and Mg the weight (both in dynes) the work done by F_1 is $F_1\ (h_1 + h_2)$, which must be equal to the energy lost. Therefore

$$F_1\ (h_1 + h_2) = Mg\ (h_1 - h_2).$$

The force F acting on the body and producing the acceleration referred to in equation (4) is $Mg - F_1$. This may be called the "reduced weight." Calculate the torque produced by this reduced weight, and substitute in equation (4). It will be necessary to measure the diameter of the drum, preferably with vernier calipers.

Find a in equation (4) by measuring with a stop-clock the time taken for 8 complete revolutions of the disc, starting from rest. Hence, find the moment of inertia I of the disc. Measure also, the time taken for the weight to descend the whole distance of 100 cm.; this is for use later on.

Carefully place the iron ring centrally on the disc, and repeat the process outlined above, using a larger weight at the end of the thread. Calculate in this case the "reduced" weight, and the moment of inertia of the combination. Find from this the moment of inertia of the ring alone. Measure the inner and outer radii of the ring (r and R), and weigh it. Compare the observed value of I with that calculated from the formula

$$M\,\frac{R^2 + r^2}{2}.$$

In the report, add a comparison of the potential energy delivered by the descending weight to the disc (in the first case only) to the kinetic energy acquired by the disc when it has its maximum angular velocity. Calculate the maximum angular velocity by equation (1), using the total time of fall as t in the formula.

SECTION II. HEAT

20. DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT

Introductory Remarks. Heat is a form of energy, and a fundamental physical law states that a quantity of heat is equivalent to a perfectly definite amount of mechanical energy. The amount of mechanical energy which is equivalent to one unit of heat is called the *mechanical equivalent of heat.* Mechanical energy can be completely converted into the equivalent heat, but no device will convert all of the heat available into its equivalent mechanical energy. This non-reversibility of the transformation is because of the fact that heat is the lowest form of energy and is the form toward which all others tend to pass.

Object. The object of the following experiment is to convert a measured amount of mechanical energy completely into heat, and to measure the resulting quantity of heat. From these determinations, the mechanical equivalent of heat can be obtained. The mechanical energy is easily measured, but the determination of the heat is difficult because of the inevitable loss of some of the heat during the experiment, which necessitates certain corrections. The corrections for the loss of heat, however, add interest and value to the experiment because the method by which the corrections are made is illustrative of much of the more advanced experimental methods.

Description of the Apparatus. In the machine for this experiment, shown in Fig. 24, a vertical spindle carries at its upper end a brass cup A. Into an ebonite ring, concentric with A, there fits tightly one of a pair of hollow truncated cones. The second cone fits into the first one and is provided with a pair of steel pins which correspond to the two holes in a grooved wooden disk B. In the experiment, the disk B prevents the inner cone from revolving when the spindle and the outer cone revolve; it is the friction between the two cones which converts into heat the mechanical energy supplied. A heavy ring, resting on the disk and held by two pins, serves to give a suitable pressure between the cones.

A brass wheel is fixed to the spindle, and by a string passing around this wheel and also around a hand-wheel, motion is imparted to the spindle. A pair of guide pulleys prevents the string from running off the wheel. Above the wheel is a screw cut upon the spindle; this screw actuates a cog-wheel of 100 teeth, which runs a cyclometer or speed counter.

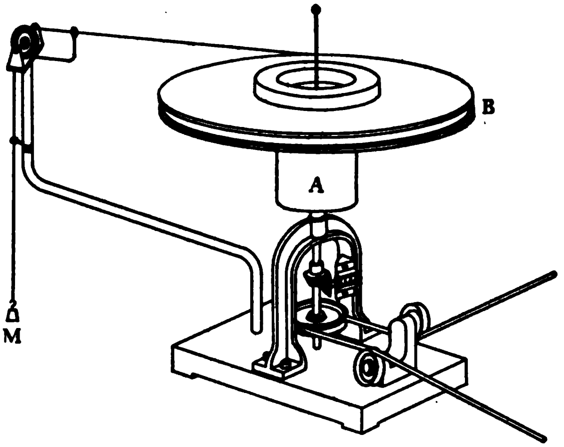

Fig. 24

To the base of the apparatus one end of a bent steel rod is attached; the rod can be fixed in any position by a nut beneath the base. The other end of the rod carries a cradle in which runs a small guide pulley, this pulley being on the same level as the disk. The cradle turns freely about a vertical axis.

A fine string (plaited silk fishing line) is fastened to the disk and passes along the groove to the point of tangency; it then passes over the pulley and is fastened to *M*, a mass of 200 or 300 gms. On turning the hand-wheel, it is easy to regulate the speed so that the friction between the cones just causes *M* to be supported at a nearly constant level.

To prevent the string from running off the guide pulley, a stiff wire with an eye is fixed to the cradle in such a manner that the eye is on the same level as the groove of the pulley and about 5 cms. from the axle of the pulley towards the disk. If the string be passed through this eye, it will always turn the cradle so that the string runs fairly over the pulley. In order to prevent the mass M from being wound up over the pulley, an eye is fixed to the steel rod, and the string supporting M passes through this eye. With these arrangements it is impossible, even with unskilful driving of the hand-wheel, either to throw the string off the guide pulley or to wind M up over the pulley.

Setting up the Apparatus. The frictional machine, or *Jouler*, as it may be called, is firmly clamped to the table, and the hand-wheel is also clamped to the table at a distance of a few feet. Care must be taken that the driving string runs properly, without any risk of slipping off the hand-wheel. The steel rod is also fixed in a convenient position.

A thermometer is hung from a support so that it passes through the central aperture in the disk, and almost touches the bottom of the inner cone. The thermometer should also pass through the hole in the stirrer.

The string supporting M should be of such a length that, when as much as possible has been unwound from the disk, M is not quite in contact with the floor.

Before putting the cones together, the rubbing surfaces must be carefully cleaned, and then four or five drops of oil must be put between them; the bearings of the spindle and guide pulleys should also be oiled.

Method of Experimenting. The cones, cleaned and oiled, are weighed together with the stirrer. The inner cone is then filled up to about 1 cm. from its top with water 2° or 3° below the temperature of the room, and the system is again weighed. The Jouler is now put into working order, one observer X taking his place at the hand-wheel, and a second observer Y at the Jouler. By working the Jouler, the water is now warmed until its temperature is as nearly as possible equal to that of the room. The observer Y stirs the water and notes the temperature every 30 seconds. After the index of the counting wheel has been read, and the temperature (θ_1) of the water has been *carefully* observed and recorded, the operator X turns the hand-wheel fast enough to raise the mass M so far that the string supporting M is tangential to the edge of the disk. If the string be not tangential, the moment of its tension about the axis of revolution is seriously diminished.

After the temperature has risen 6° or 8°, the motion is stopped, and the highest temperature (θ_2) shown by the thermometer is *carefully* read. This maximum temperature will be reached several seconds after the Jouler is stopped, so that stirring should be continued until this observation is made. The final reading of the counting device is also observed. From this reading, together with the initial reading, the total number of revolutions n of the spindle can be ascertained.

Observations to determine the correction for cooling are then made. Without disturbing the apparatus, the temperature of the water is raised about 2° above θ_2 by working the Jouler. The Jouler is then brought to rest, and the water is allowed to cool, with frequent stirring, and observations of the temperature are taken at intervals of about one minute for 10 or 15 minutes.

Calculation of the Correction for Cooling. Since, for small differences of temperature, Newton's Law of cooling is nearly true (*i. e.* rate of cooling is proportional to temperature difference), and since the temperature of the water rises nearly uniformly, the correction ϕ to be added to $\theta_2 - \theta_1$ is approximately equal to half the product of the rate of cooling at the temperature θ_2 and the time occupied by the rise $\theta_2 - \theta_1$.

Calculation of the Work Done. When the spindle has made n turns, the work spent in overcoming the friction between the two cones is the same as would have been spent if the outer cone had been fixed and the inner one had been made to revolve by the descent of the mass M grams. In the latter case M would have fallen through $2\pi nr$ cms., where r is the radius of the groove of the wooden disk. Hence the total work spent in overcoming friction and thus producing heat is $2\pi nrMg$ ergs.

Calculation of the Heat Produced. Let W grams be the mass of the water, and w grams the mass of the cones and stirrer.

The specific heat of the metal may be taken as .095, and thus the system of cones, stirrer, and water is thermally equivalent to $(W + .095w)$ grams of water.

During the action of the Jouler the heat in the system of cones, stirrer, and water has increased by $(\theta_2 - \theta_1)(W + .095\,w)$ units, while $\phi\,(W + .095\,w)$ units have escaped from that system by radiation, conduction, and convection. Hence the total number of thermal units produced is $(W + .095\,w)\ (\theta_2 - \theta_1 + \phi)$ water-gram degrees.

Calculation of the Mechanical Equivalent of Heat. If J denote the number of ergs of work which must be spent to raise the temperature of one gram of water by one degree, we have

$$J = \frac{2\pi nrMg}{(W + .095w)\ (\theta_2 - \theta_1 + \phi)}.$$

21. HEAT OF COMBUSTION OF GAS

Introductory Remarks. It is often important to know the " heat of combustion " of a fuel, i. e., the heat derived per unit of fuel burned. Its numerical value depends, of course, on the units in which it is expressed. The heat may be given in calories, in British Thermal Units (B. T. U.), or in any other energy or work unit, while the amount of fuel may be either by weight, by volume, or even by cost. The heat of combustion is to be found in this experiment for ordinary illuminating gas, and in the second part of the experiment, the efficiency of an ordinary sort of cooking apparatus is to be tested.

Apparatus. The calorimeter is of a special type, supplied with a continuous flow of water. The hot gases resulting from the flame pass out through pipes surrounded with water, where they give up their heat. The rate of gas flow is measured by a special type of gauge, in which the motion of an indicator on a scale furnishes directly the number of " cubic feet per hour " of gas which is being consumed. In the second part of the experiment a small gas stove and a cooking vessel are furnished. Appropriate thermometers are supplied for both parts.

Method. *First part.* Arrange a small but steady flow of water through the calorimeter, and a steady gas flame, burning blue without any white in it. Put the burner under the calorimeter, but *not* before the water is started. Adjust the gas current by the gauge at 4 or 5 cubic feet per hour, and keep it constant at the chosen point thereafter. Note the readings of the inlet and outlet thermometers of the calorimeter, and wait until these temperatures have become steady. Then read both thermometers to the smallest fraction of a degree that is possible. At the same time catch the water stream in a vessel (previously weighed) for a measured time interval — time enough nearly to fill the vessel; weigh the water in the vessel, and hence find the quantity of water flowing through the calorimeter per second.

Calculate the heat received (in calories) by this water per second, and from this find the heat of combustion of the gas in calories per cubic foot, and also in calories per cent, assuming the gas to cost $1.00 per 1000 cubic feet.

Second Part. As a test of a common type of cooking vessel, and of a gas stove, put a pint or so of water in the vessel, weigh it, and heat it with a measured fuel supply for a measured time. It should be warmed through 50 or 60 degrees. The useful heat for cooking purposes is merely that which goes into the water, so that no allowance need be made for the heat absorbed by the vessel itself.

Calculate the heat received by the water in this time, and also the heat given out by the gas consumed (using data from *First Part*) and hence find the efficiency of the cooking outfit.

22. HEAT CONDUCTIVITY OF COPPER

Object. To measure the heat conductivity of copper.

Apparatus. A copper bar, imbedded in heat-insulating material, except at its ends, one of which is heated by steam, and the other cooled by running water; thermometers; gauge for indicating the rate of flow of the water; steam apparatus.

Method. Different materials have different powers of conducting heat. In order to compare these, a coefficient is determined for each, which is called the heat conductivity; it expresses the amount of heat, in calories, transmitted in one second across a plate 1 cm. thick, and having a face of 1 sq. cm. area, when the opposite faces differ by 1° C in temperature. If this coefficient be called k; and if the ends of a bar of the material are kept at different temperatures, t_1 and t_2 then a quantity of heat Q will be transmitted through the bar per second such that

$$Q = kA \cdot \frac{(t_2 - t_1)}{d}$$

where A is the area of cross-section of the bar, and d is its length.

In this apparatus, a bar of copper is used, whose diameter is 3.80 cm., and there is a length between the thermometers of 20.3 cm. The thermometers rest in little holes in the copper bar. One end of the bar is heated by steam, and the other is cooled by running water. The temperature of the running water can be measured as it enters the apparatus, and again as it leaves, using for this purpose two rather sensitive thermometers. The stream is to be kept constant. No particular rate of flow is required, but the inlet and outlet temperatures should differ by not less than two degrees, nor more than five.

The steam apparatus must be supplied with plenty of water, and then heated. After steam has begun to flow into the conductivity apparatus, the water stream is to be started, and thereafter kept constant. The rate of flow is to be measured by catching a quantity of water in a vessel and weighing it, allowing it to flow into the vessel for a measured time. This should be done more than once. Hence the number of grams of water flowing through the apparatus per second is obtained. The inlet and outlet thermometers give two temperatures, the difference of which enables us to calculate the quantity of heat flowing into the hot end of the bar per second.

The other two thermometers must be read at the same time, after the flow of heat has become steady, and these give t_2 and t_1 in the formula above. Hence all the quantities above are known, and the conductivity may be calculated.

If there is time, all the measurements should be repeated, using a different rate of flow of water. Average the two values obtained for k.

What causes for error should affect the accuracy of the result, and will the result be likely to be too high on this account, or too low ?

23. EXPANSION OF LIQUIDS

Object. (*a*) To measure the expansion of mercury, and (*b*) to study the changes in volume of a quantity of water through a range of temperature from 0° C to about 10° C.

Apparatus. (*a*) Two tall tubes containing mercury, connected at the bottom, one heated by a current of steam, the other kept near room temperature by circulating water; thermometer; level.

(*b*) An ordinary mercury thermometer, and another one like it, except that the mercury has been replaced by water; can; cracked ice, or snow; stirrer.

Method. (*a*) If two different liquids, which do not mix, are connected together in a U-tube, they will stand at different levels in the two sides, since a longer column of the lighter liquid is required to balance the heavier. The lengths of the two columns will be in the ratio of the volumes of the liquids per gram, or the inverse ratio of their densities. In this experiment, the two liquids are each mercury, but at different temperatures, and the warmer column is the lighter. If L and V are the length and volume respectively of the cold column, and $L + l$ and $V + v$ corresponding terms for the hot, then

$$\frac{L+l}{L} = \frac{V+v}{V} \quad \text{or} \quad \frac{l}{L} = \frac{v}{V}.$$

If the difference in temperature is t degrees, the coefficient of expansion is v/Vt, which is the same as the increase in volume of 1 cc. of the liquid when it rises 1° in temperature. To measure this coefficient, we must get l, L, and t.

L may be measured with sufficient accuracy by means of a meter bar, from the middle of the connecting tube at the bottom, to the top of the surface of the cold column. The difference in temperature is readily found, the temperature of the steam being taken from a table of boiling points, after the barometric height has been measured. The temperature of the cold column is to be measured by lowering a thermometer into the mercury from the top. The increase in height, l, is found by means of a couple of moving pieces near the upper surface of each tube. These carry forked extensions which partly enclose the mercury tubes, and by moving these up or down, the front and rear branches of the fork may be brought exactly in line with the *top* of the mercury surface inside the tube. This is to be done on each side, but, note that the holder which carries these forked pieces must be kept level, and has an adjusting screw for this purpose. When both markers have been properly set, the vernier attached to one of them is to be read against the scale held by the other. The reading is to be obtained after the heating has continued so long that the mercury column has ceased to expand; it gives the expansion l needed for the calculation of the coefficient.

In the report on this experiment, explain clearly why the expansion of mercury could not be obtained as accurately, and more easily, by having only one tube, closed at the bottom, and measuring the change in height of the mercury in it.

(*b*) The change of volume of the water in the thermometer alters the height of the thread of water in the stem. These changes would indicate real changes in volume in the water, if it were not for the fact that the glass also expands. If the rate of expansion of the water and the glass were equal and opposite, evidently no change would occur in the height of the water column. This will be found to be the case at one particular temperature, but at no others. To allow for the expansion of the glass, and obtain that of the water alone, we treat the observations in the manner explained below.

The necessary measurements consist merely of simultaneous readings of the two thermometers at about ten intervals between 0° C to 10° C. For 0° C, make up a dish of ice-water consisting chiefly of ice, and leave the two thermometers in it long enough to make sure that their temperature is uniform and steady. For the higher temperatures, remove the ice, and stir vigorously, so as to keep the temperature uniform throughout the vessel. See that the temperature is changing very slowly, if at all, when readings are being taken. Add small quantities of ice (and stir) if necessary to prevent rapid changes.

The results are to be plotted as a curve showing how the readings on the water thermometer changed with the temperature. If the mercury thermometer did not read zero when in melting ice, its reading then is an error, and is to be applied as a correction to all other temperature readings. This curve shows the difference in expansion between water and glass, that of glass being practically constant throughout. When the water thermometer was first made it was filled with mercury and then graduated. Since the coefficient of expansion of mercury is .000181, and of glass .000025, the difference, or .000156, represents the relative coefficient of mercury and glass, and this means that the apparent increase in the volume of the mercury for each degree is this fraction of its total volume. The volume of the fine capillary tube contained in the space between two degree marks will be called the volume of one degree-division, and denoted by v. If the volume of the bulb at 0° C is V, then the above statement means that

$$\frac{v}{V} = .000156.$$

Suppose that the volume of the glass bulb (V) is just equal to that of the water when the temperature is 0°. Then at 1°C the volume of the bulb will have become $V + .000025V$, and the apparent volume of the water will be too small (because the expansion of the bulb has let the water down into it) by an amount $.000025V$. But, the volume of one degree-division is $.000156V$. Hence, if we correct the observed position of the water surface at 1°C by adding to it the fraction $\frac{.000025}{.000156}$ or .16 of a degree-division, we shall obtain what would have been the height of water in the tube, if the glass had remained unchanged in volume. Similarly, at 2°C we must add to the observed water height an amount $2 \times .16$ of a degree-division; or at $t°$ C, add $t° \times .16$ degree-divisions. To do this properly, we should first draw our observed curve as smoothly as possible, using a fine sharp pencil point, and then take off from this curve the water readings corresponding exactly to each degree of temperature. Add to these the necessary corrections, and replot the new values on the same sheet. Draw a second smooth curve through these points.

From the second curve, deduce the temperature at which water has its greatest density.

SECTION III. SOUND

30. PITCH BY THE MONOCHORD

Introductory Remarks. Both from a physical and a musical standpoint the pitch of a note is its most important characteristic, and depends upon the number of vibrations per second. If the sounding body is a stretched wire (or string), the number of vibrations per second, when the wire is plucked transversely, depends upon its mass, its length, and the tension with which it is stretched, and can be expressed exactly by a formula involving these quantities. The vibration of a string, therefore, furnishes a very accurate and convenient method of determining the pitch of a note.

A series of eight notes whose pitch numbers bear to each other certain simple ratios forms the musical scale. These ratios determine what are called the musical intervals of the notes. When the musical interval of two notes is simple, that is, when the ratio of the pitch numbers is a simple ratio, as 2 : 1 or 4 : 3, etc., the notes, when sounded together, give a pleasant sensation, and are said to be in harmony or in accord. The eight notes constituting the scale are denoted, beginning with the lowest, by the letters *C*, *D*, *E*, *F*, *G*, *A*, *B*, *c*. The musical interval of the eighth note to the first, *c* to *C*, is called an octave; the interval of the fifth to the first, *G* to *C*, is called the interval of a fifth; of *E* to *C*, the interval of a third; and so on.

Object. The object of the following experiment is to determine the pitch numbers of some of the notes in the musical scale, and to obtain the ratios corresponding to an octave, a fifth, and a fourth.

Apparatus. The monochord, as shown in Fig. 25, will be used to determine the pitch numbers of four tuning forks, and consists essentially of a steel wire stretched over a resonating chamber. The wire passes over a movable fret and a right angle lever which rests in a *V* slot on the end of the box. The right angle lever is shown enlarged in Fig. 26. On the end of the wire is a bag for holding weights.

A set of four tuning forks is provided, the pitch numbers of which form a major chord.

Discussion. If L denotes the length in centimeters of the vibrating part of the wire, m the mass in grams of each centimeter, and t the tension expressed in dynes, then n, the number of complete vibrations per second, is given by the formula,

$$n = \frac{1}{2L}\sqrt{\frac{t}{m}}$$

Since L, t and m can be readily determined, n, the pitch of the note, can be calculated.

In order, therefore, to measure the pitch of any note, it is only necessary to vary the length of the monochord wire until the note and the sound from the vibrating wire are in unison.

When two notes nearly, but not quite, in unison are sounding together, they at one moment re-enforce and at the next oppose each other; the sound is alternately loud and faint. These pulsations of the sound are technically called *beats*, and furnish a mechanical method of ultimately securing unison. The beats cannot be detected until the wire and the fork are nearly in unison, but when once they are obtained, the length of the wire should be varied in such a direction as to make the beats succeed each other more and more slowly, until ultimately indistinguishable. If the fork is not mounted on a resonant box, its sound may be greatly re-enforced by holding its shank on the monochord. In this case the beats may be felt by resting the fingers lightly on the box.

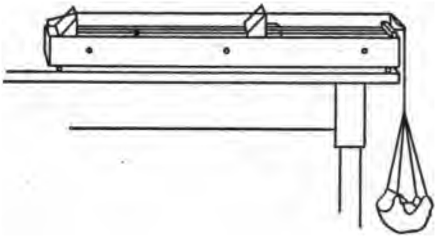

FIG. 25

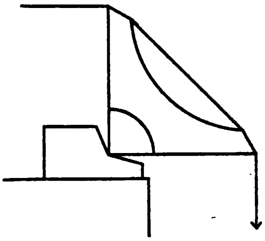

FIG. 26

Procedure and Results. Weigh a steel wire about a meter and a half long, measure its length, and calculate the mass m per centimeter. Stretch it over the monochord by a load of eight or ten kilograms, and tune to unison with a tuning fork, varying the length by means of the movable fret. When in unison, measure the length L of the vibrating part of the wire. Disturb the setting and make several independent determinations of the same length, and find their average.

If the wire on the monochord is stretched over a right-angle lever (see Fig. 26), in order that the tension in the vibrating part of the wire may equal the load, it is necessary that the vertical and horizontal arms should be of equal length. They are, probably, nearly but not exactly of equal length. The error arising from slight inequality may be eliminated by using in the calculation of n the mean of the average of the length L as obtained above and the average of several independent settings when the lever is turned over so that the arm which was before horizontal is now vertical. When the longer arm of the lever is horizontal, the tension in the vibrating part is greater than the load; when, however, the longer arm is vertical, the tension is less. Care must be taken throughout the experiment that the lever does not rock so far forward or backward as to rest against one of the shoulders of the recess in which it plays.

Calculate in dynes the tension with which the wire is stretched, including as part of the load the weight of the pad holding the weights. With the mean of the two averages of the length of the wire as determined above and the values of m and t, calculate the pitch number n of the note emitted by the wire by means of the above formula. This will also be the pitch of the fork, since the two are in unison.

Also determine the pitch of the same fork, using a different stretching load on the wire.

The fork should be set in vibration by bowing, or by striking with a rubber hammer. If the fork is struck against a hard surface, it is liable to become worn and its pitch changed in consequence.

Following the method outlined above, determine the frequency of vibration, n, of each of the four notes, using as a load about 8 to 10 kilos. Having determined the pitch number of each, calculate the ratio expressing the musical interval between each note and the lowest note in the scale, C. By an examination of these ratios, determine to what simple ratios of whole numbers the interval of the octave, the fifth and the fourth, may be reduced. In making the reduction, consider one vibration in the pitch number as a possible experimental error.

31. VELOCITY OF PROPAGATION OF SOUND

Introductory Remarks. Sound is transmitted as longitudinal vibrations, with a definite velocity depending upon the elasticity and density of the medium. One method of measuring the velocity of propagation of sound is by means of the resonance tube, which is a device providing an adjustable column of air. The resonance tube method will be used in the following experiment, and the work is of interest, not entirely because of the results obtained, but because of the physical phenomena and principles involved in the method.

Object. The numerical result to be obtained in the following experiment is the velocity of sound in air at 0°C, as determined by the resonance tube method.

Apparatus. The resonance tube which is to be used consists of a brass tube about four feet long provided with a movable but tightly fitting piston. At least two tuning forks of different pitches will be used as sources of sound.

Discussion. One way of explaining the action of a closed resonance tube is as follows: Suppose a fork, whose pitch number is n, be caused to vibrate near the open end of the resonance tube. If now, starting with the piston near the open end, the length of the column of air be slowly increased, a position will be found for which the intensity of sound is greatly increased. This increase in sound results from the resonant vibration of the column of air. The impulse sent down the tube by the forward vibration of the fork, travels the length L_1 of the tube, and is reflected back in time to re-enforce the fork in its backward motion. Thus during the forward motion of the fork, that is, during half a vibration, the sound travels twice the length of the tube, $2L_1$. The distance actually traveled by the impulse in going each way is a little greater than the length of the tube on account of the reflection from the sides and the spreading at the open end. The correction to be added is equal to nearly one quarter the diameter d of the tube. During half a vibration, therefore, the sound goes $2\left(L_1 + \frac{d}{4}\right)$ centimeters, and hence during n whole vibrations, or one second, the sound goes $4n\left(L_1 + \frac{d}{4}\right)$ centimeters; whence the velocity of sound in air at the temperature of the room is $V = 4\,n\left(L_1 + \frac{d}{4}\right)$. If the length of the air column be still further increased, a second position of the piston will be found for which the sound will be greatly re-enforced. By reasoning similar to the above, it may be shown that the impulse now travels the length of the tube and back in time to re-enforce not the next, but the next but one backward motion of the fork. The sound, therefore, travels from the first resonance position of the diaphragm to the second resonance position and back again to the first position during one vibration. Let this distance, over and back, be denoted by $2L_2$. During n vibrations, or one second, it would travel n times as far. The velocity of sound is therefore also given by the relation.

$$V = 2\,n\,L_2.$$

In like manner, if the tube is long enough, a third, fourth, etc., position of the piston can be found which will give resonance.

For other intermediate positions of the diaphragm, slight resonance of the overtones of the fork may be obtained, but they can be distinguished by their higher note.

While the air in the longer resonance tube is vibrating with the fork, the waves reflected from the closed end of the tube interfere with the succeeding waves coming down the tube at the first position of the diaphragm, causing the air here to remain almost at rest. This point is called a node. The air in the rest of the tube is in vibration to and fro, at a maximum midway between the two positions of the diaphragm and at the open end.

The velocity of sound in air is somewhat affected by the moisture present. The change in velocity of sound due to this will probably not amount to half of one per cent of the whole velocity.

The method outlined above gives the velocity of sound at the temperature at which the observations were taken. The velocity of sound increases about 60 centimeters for every centigrade degree rise in temperature. The exact formula, giving the velocity of sound at 0°C, from the velocity at t degrees, is as follows:

$$V_0 = \frac{V_t}{\sqrt{1 + .00366\,t}}.$$

Procedure. Obtain the length of the air column for the first, second and, if possible, the third positions of the piston which give maximum resonance.

In this experiment, as in several others, the settings for obtaining the data are somewhat indefinite. One setting might, therefore, be far from the true value. It is consequently necessary to make many settings with the expectation that the average of many indefinite observations will have a much smaller error. In this particular case, at least 6 or 8 observations for each length should be made.

Furthermore, in making each observation a much better setting will be obtained if the piston be rather rapidly moved to and fro through the position of maximum resonance, diminishing the extent of the excursions each time, and finally settling down to what is thought to be the position which gives the maximum intensity of sound.

Repeat the process outlined above for both of the forks.

Record the temperature of the room and the inside diameter of the resonance tube.

Results.

1. Calculate the average velocity of sound at room temperature from the data for the first and second, and also for the second and third positions of the piston, for each fork.
2. Reduce each of the above average velocities to the velocity at O°C, and average these two values.

SECTION IV. ELECTRICITY

41. ELECTROSTATICS

Object. To study some of the phenomena of electric charges.

Apparatus. Electroscope with plate on top; small extra plate to fit the electroscope; electrophorus, consisting of hard rubber base and metal plate with insulating handle; Leyden jar; metal can on insulating base; mica; wire; fur; woolen cloth; silk; rods of hard rubber and glass.

Method. Various simple experiments on electrostatics are to be performed as indicated in the sections below. Each experiment is to be carefully, though briefly, explained; simple sketches may be used to advantage to shorten the explanations. The sign of the charge on each part of the apparatus should be shown, and the process identified by which this charge was formed.

(*a*) " Earth " the case of the electroscope by connecting it by a wire to the gas pipe and keep it so throughout the whole experiment. Charge the hard rubber rod by rubbing it with fur, at some distance from the electroscope, and then bring the rod near the plate, from above; then remove it. Two quite different effects may occur, depending on how highly charged the rod is, and how near it comes to the plate. Vary *both* these conditions widely, including as one case actually touching the rod to the plate when feebly and when highly charged. In each trial, the electroscope leaf is to be discharged at first. If this is not the case, a touch of the finger to the plate will discharge it. Describe what different things happen in all these cases, and why. In your explanation make use of the word "ion".

(*b*) Bring the charged rod about an inch above the (discharged) electroscope. Touch the plate with the finger; then remove first the finger, then the rod. Describe and explain.

(*c*) Leaving the electroscope charged, as in (*b*), charge a glass rod by rubbing it with silk, and approach it to the plate, watching the leaf. Describe and explain. Compare this with approaching the rubber rod, charged by fur. Compare also the effect obtained by rubbing the glass rod on fur, and testing it in the same manner.

(*d*) Charge a rod highly, and then pass it for a moment through the gases rising (say a foot) above the flame from a bunsen burner. Test the rod for charge. Recharge the rod highly, wipe thoroughly with the hand, and test for residual charge.

(*e*) Rub the plate itself of the (discharged) electroscope with fur. Test the sign of the charge on the leaf by bringing up a charged rubber rod.

(*f*) Place the can on its insulating base, and connect it by a wire with the plate of the electroscope. Put the fur inside the can, and rub it with the (discharged) hard rubber rod while it is there. Watch the leaf. Remove the rod and note the effect. Test the sign of the charge on the leaf. Repeat, removing the fur instead of the rod. How does this experiment prove that equal quantities of opposite charges are produced by friction ?

(*g*) Put a charge on the electroscope by induction, as in section (*b*). Lay a thin piece of mica on top of the plate. Holding the small extra plate in your hand, bring it down from above, keeping it parallel to the plate of the electroscope, and approaching it; let it finally rest on the mica. Watch the leaf during this process, and while the extra plate is lifted off again. In your explanation of this section, and of the one following, make use of the terms potential and capacity.

(*h*) Connect the outer coating of the Leyden jar to the gas pipe, and the inner to the plate of the electroscope. Place the electrophorus at some distance from the electroscope, and rub the hard rubber base plate of the electrophorus with fur, so as to charge it highly. Holding the upper plate of the electrophorus by its insulating handle, bring it down on top of the base. Touch the upper plate a moment with the finger while it is there, and then lift the plate off and bring it up to the rod which is connected to the inner coating of the Leyden jar. Observe the deflection of the leaf after the spark has passed to the jar. Repeat this process, bringing up the charged electrophorus plate to the jar until the leaf rises no higher. Why does the spark cease to occur between the jar and plate ? How many times was it necessary to bring the charged plate to the jar before maximum deflection of the electroscope leaf was obtained ? Disconnect the electroscope from the jar, and determine the number of times the charged electrophorus plate must be brought near the electroscope plate to produce maximum deflection.

Is the charge on the base of the electrophorus materially reduced by the use of this instrument? Where does the energy come from which appears in the form of electric charge when the electrophorus is operated?

(*i*) Prepare a brief and compact summary in your report, giving a list of the facts or laws of static electricity which you have derived from this experiment.

42. EQUIPOTENTIAL LINES AND LINES OF FLOW

Flow of Electricity through Extended Conductors

Introductory Remarks. A current of electricity which passes through a thin conducting sheet of metal or liquid spreads throughout the conductor, forming what is termed a current sheet. The density of the electric current at different points in the conducting sheet is obviously different; in general, the density being greatest in that region of the conductor which lies in the shortest path between the electrodes which lead the current into and out of the conducting sheet.

Two points in the conducting sheet are said to be at the same potential when no current will pass through a conductor, as for instance a wire, which connects the two points outside the conducting sheet. The locus of all points at the same potential is called an *equipotential line.*

A *line of flow* is a line representing the complete path of a filament of current as it passes from one electrode to the other. The direction of the lines of flow, therefore, shows the direction of the current at every point in the conducting sheet.

A few general propositions may now be stated.

(1) Since the potential changes most rapidly in the direction perpendicular to the equipotential lines, and since the current will always flow in the direction of greatest change in potential, the equipotential lines are everywhere perpendicular to the lines of flow.

(2) In those regions where the equipotential lines are nearest together, the rate of change of potential with distance is greatest and the current density is greatest.

(3) At the boundary between the conductor and an insulating medium the lines of flow are parallel to the boundary line, and hence the equipotential lines are perpendicular to the boundary.

(4) If any part of the conducting sheet has a different conductivity, produced, for instance, by a greater or less thickness of the sheet, or by the presence of another material of different resistance, the lines of flow and the equipotential lines will be distorted.

(5) If any part of the conducting system has a negligibly small resistance compared with the resistance of the rest, there is no change in potential throughout the better conductor, and the boundary lines are equipotential. Consequently, the lines of flow meet normally the boundary between the assumed perfect conductor and the surrounding conducting sheet.

The above laws may be extended practically without change to three dimensional problems in current flow. Furthermore, if the lines of forces in an electrostatic field about charged conductors, or in a magnetic field about magnetic poles, be mapped out, the drawing would appear exactly similar to the lines of flow about a like configuration in the flow problem. The lines of force and the equipotential lines or surfaces for the electrostatic and magnetic fields will likewise conform to the general laws given above.

The maps obtained in this experiment for various cases of current flow are of considerable theoretical interest. Furthermore, there are many times in practice where the knowledge of the flow of earth currents, the field of force about dynamo fields, etc., are of great importance, and the experience gained in performing an experiment such as this enables one to predict to some extent the results in any particular problem.

Object. The object of this experiment is to give a general knowledge of the laws governing the flow of current through extended conductors.

Apparatus. The conducting sheet in this experiment is a thin sheet of water contained in a square tray, which has on its bottom a coördinate system in order that the points may be read off and transferred to another similar sheet of coördinate paper. An intermittent or alternating current of a frequency of several hundred per second is caused to flow through the conducting sheet. This current may be obtained by means of an induction coil, a small alternator of 50 to 100 volts, or by interrupting the current from the 110 volt direct current line after passing through a high resistance which serves to protect the apparatus. Electrodes for leading the current into and out of the water, are provided, and may be placed at any point in the tray. Variously shaped conductors and insulators may be used for making up several configurations. A telephone receiver and exploring electrodes are provided for finding the points which are at the same potential.

Procedure. Fill the tray to a depth of about 1/4 inch of water containing a trace of salt, and level carefully. Connect the source of current to the electrodes provided for the purpose, hereafter called "current electrodes." Connect one terminal of the telephone receiver to an electrode to be used as the stationary exploring electrode, and the other terminal to the movable exploring electrode.

Case 1. Place the two current electrodes at two symmetrically opposite points on the principal axis of the conducting sheet as, for instance, at + 8 and − 8. Place the stationary exploring electrode at say + 7, and with the movable exploring electrode determine, by noting at what points there is silence in the receiver, enough points on the equipotential line to enable the complete curve to be drawn. In this way, determine the equipotential lines which pass through the points +6, down to 0 on the horizontal axis. Since this particular arrangement is symmetrical, the other half of the map need not be experimentally determined. It may be necessary, in order to determine completely the shape of the field at every point, to obtain more equipotential lines than are suggested above.

More complicated cases may now be arranged by using variously shaped conductors instead of points for electrodes, and by placing between the electrodes bars or rings of metal, or insulators. There follow several suggested cases which have either theoretical or practical application, but the student is at liberty to make up cases of his own. For a two-hour laboratory period it is suggested that two cases be studied (*a*) with no obstruction between the current electrodes, (*b*) with a metal bar laid down symmetrically, but at 45° to the line between the current electrodes.

If there is time, one or more of the following cases should also be investigated.

(1) Arrange a closed ring of metal between two point electrodes, or between a point and a bar electrode, or between two parallel bar electrodes. The inside of the ring should also be explored. This case illustrates the shielding action of a complete conducting surface for the region inside, a principle of great importance in protecting galvanometers and electrostatic instruments from outside magnetic and electrostatic fields, respectively.

(2) Substitute an insulating ring or disk for the conducting ring in (1).

(3) Place two ring electrodes of different sizes at symmetrical points on the axis. Show that the equipotential lines and lines of flow (lines of force), outside the rings, are the same as for two point electrodes placed at definite points inside the rings.

(4) Place one point electrode inside a metallic ring between the center and circumference. Place the other point electrode outside about 5 cm. from the side of the ring in a direction farthest from the first point electrode.

(5) Place a ring, from which a piece has been cut, on the axis between point electrodes, the hole in the ring facing and rather near to one electrode.

Results. The results comprise maps for as many cases as can be done, showing the equipotential lines and lines of flow. The maps obtained are to be used in verifying the general proposition given in the introductory remarks. Conclusions and remarks should be recorded.

The maps should be drawn on ordinary cross-section paper on half the scale of the original measurements, and should be complete, *i. e.*, should show the entire tray, and the arrangement of equipotential lines and lines of flow in it for each case.

GALVANOMETERS AND AMMETERS

Galvanometers are instruments for detecting small currents of electricity, and may or may not be used to measure the strength of the current.

Ammeters are less sensitive galvanometers made in portable form and used to measure the strength of the current passing through the instrument.

Since galvanometers and ammeters will be used in many of the following experiments, a brief description of their construction is given below.

The effect which causes the deflection in practically all galvanometers is the force exerted between one or more wires carrying a current and a magnetic field. There are two main types of galvanometers according to whether the magnet which supplies the field, or the coil carrying the current, is the moving system.

In the first type or *moving magnet* form of galvanometer, a light magnetic needle is acted upon by the current circulating in one or more stationary coils of wire. The tangent galvanometer is of this type, and consists of a single short magnet suspended or pivoted at the center of a large ring-shaped coil. Although insensitive, this instrument is useful for measuring a current in terms of the angle of deflection, the dimensions of the instrument, and the strength of the earth's magnetic field. Such an instrument is used as a primary instrument for determining absolutely the value of an unknown current, and is never used solely as a detecting instrument. If the coils of wire are brought very near to the needle and are wound with many turns of fine wire, the moving needle instrument can be made the most sensitive of any type. This form of instrument is often called a Thomson galvanometer.

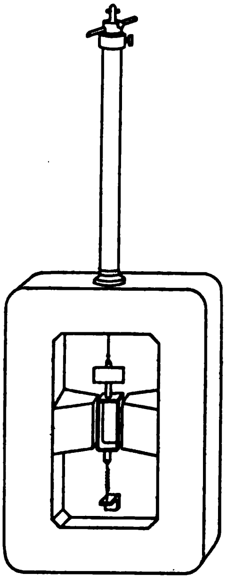

FIG. 27

The *moving coil* or *d'Arsonval galvanometer* is the most common type and consists essentially of a coil of fine wire pivoted or suspended in a powerful magnetic field. In the most sensitive instruments, the coil is suspended by a very fine wire or ribbon of metal, which also serves to lead the current into the coil, the current leaving the coil by a spiral wire or ribbon below. Fig. 27 shows one form of the d'Arsonval galvanometer used in the laboratory.

The rotation of the moving system of the sensitive galvanometers is measured by means of a telescope which views the image of a stationary scale reflected in a small mirror attached to the moving system of the instrument.

Most ammeters are forms of d'Arsonval galvanometers made for rough portable use. The moving coil is usually pivoted and controlled by means of two spiral hair-springs, which also serve as electrical connections to the coil. A fine pointer carried by the coil takes the place of the mirror and scale.

43. RESISTANCE BY WHEATSTONE BRIDGE

Read section on *Galvanometers and Ammeters.*

Introductory Remarks. Electrical resistance is one of the most important of the data used in the solution of all kinds of electrical problems. A knowledge of the methods of determining this factor is, therefore, of certain practical interest.

The measurement of the resistance of an electrical device or circuit may be accomplished in three ways. The *first* way of measuring the resistance is by the use of Ohm's law. If the current in the resistance and the difference of potential across the terminals of the resistance to be determined, be measured by means of an ammeter and a voltmeter, respectively, the resistance can then be calculated. The *second* method consists in measuring the current in the circuit under test, and the amount of heat produced by the current. Since Heat $= I^2Rt$, the R can be calculated. This method is evidently of very limited application. The *third,* and by far the most accurate, method of measuring resistance is by means of some form of bridge. There are two distinct forms of the bridge, namely: the slide-wire bridge and the box form. The underlying principles are the same in both.

Object. The primary object of this experiment is to give practice in the use of the Wheatstone bridge. The bridge will be used for the measurement of several resistances singly, and in parallel, and the accuracy of the latter will be checked by calculation of the parallel resistance of each combination from the single resistances.

Apparatus. The apparatus for this experiment comprises a slide-wire bridge, a sensitive galvanometer, a double key, and a dry cell. Several numbered resistances will be supplied for use with the bridge.

PRINCIPLE OF THE BRIDGE

If part of a circuit carrying a current is divided, there is a continuous fall of potential on both branches from the point at which they separate to the point at which they reunite. For every point on one branch a corresponding point on the other may be found having the same potential. If two such points *m* and *n* (see Fig. 28) be connected by a wire, no current will flow, and if in this circuit a galvanometer *G* be inserted, its needle will not be deflected. Denote the resistance of the various parts by the letters *a*, *b*, *x*, and *r*. By Ohm's law, the current in any part is equal to the difference of potential divided by the resistance, and as no current is carried off by the cross-wire, the current in *b* is equal to that in *a*; hence

$$\frac{\text{the fall of potential in } a}{a} = \frac{\text{the fall of potential in } b}{b}$$

or

$$\frac{\text{the fall of potential in } a}{\text{the fall of potential in } b} = \frac{a}{b}\,.$$

Similarly in the other branch, the current in *r* is the same as in *x*, and hence

$$\frac{\text{the fall of potential in } x}{\text{the fall of potential in } r} = \frac{x}{r}\,.$$

Since *m* and *n* are at the same potential, the fall in *a* is equal to the fall in *x*, and the fall in *b* equals the fall in r. Hence the first members of the last two equations are equal numerator to numerator and denominator to denominator. The second members are therefore equal, giving

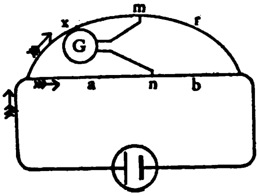

FIG. 28

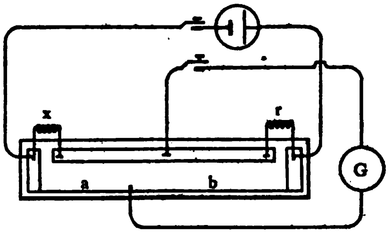

FIG. 29

$$\frac{x}{r} = \frac{a}{b} \text{ or } x = \frac{a.r}{b}\,.$$

If the resistance *r* and the ratio $\frac{a}{b}$ are known, *x* can be calculated. In determining the value of an unknown resistance by means of a bridge, either the ratio $\frac{a}{b}$ or the standard resistance *r* must be varied until the above relation is fulfilled.

The slide-wire bridge shown in Fig. 29 makes use of the first method of obtaining balance. The branches *a* and *b* are the two parts of a uniform straight wire, the ratio of the lengths of which can be varied by a sliding contact. Since the resistance of a wire is proportional to its length, the ratio of the lengths of a and b is equal to the ratio of their resistances. The resistance of r is the standard in terms of which the unknown is measured and is usually variable. A certain amount of adjustment of r is necessary because the highest accuracy of measurement is realized when the balance point is near the middle of the slide wire. The slide-wire bridge is particularly adapted to the measurement of low resistances.

In the box form of bridge, used in the next experiment, the ratio a to b is definitely fixed by known resistances to any desired ratio, 1 to 10, 1 to 100, or 1 to 1000 at pleasure, and the resistance r is then varied until balance is obtained. The variable part r of the bridge is usually adjustable in steps of one ohm from 1 to 10,000 or more, and by the proper choice of the ratio $\frac{a}{b}$ any resistance either high or low can be measured. The box bridge is, however, especially adapted to the measurement of high resistances.

Box bridges vary greatly in the grouping of their resistance coils; but, by first drawing a diagram of the ideal bridge, as above, and also of the resistance box, we may proceed as follows to complete the connections. Choose for a and b that portion of the box, the coils of which progress decimally, 1, 10, 100, etc., and for r usually the remainder of the box having the greatest range and capable of the most gradual variation. Consult before each step the ideal diagram, Fig. 28; a and b, if not already connected, should be joined by a plug if possible, if not by a short heavy strip of copper; join one end of r to b; to the other end of r join x, and the other end of x to a; where a and b join, attach one terminal of the battery, and proceeding in this way, complete the connections.

When measuring a resistance, the approximate value of which is not even known, always start with a ratio of 10 to 10, never 1 to 1, and then determine what resistance, when unplugged in the rheostat or variable part of the bridge, gives an approximate balance. This unplugged resistance is, of course, the approximate resistance of the unknown. Now choose the ratio resistances so that the unplugged resistance in the variable part will give a sufficient number of figures to secure the desired accuracy. An accuracy of 1/10% should be aimed for. As an example, suppose the unknown resistance were found, by the first test, to be about 170 ohms. In order to give the desired accuracy, the resistance unplugged in the variable part of the bridge should be in the thousands, thereby giving at least four figures in the result. That is, the ratio should be 10 to 100 in this case, and the variable resistance might be found to be 1734 ohms. If necessary, interpolate. It is clear that the ratio may be either 1 to 10, 10 to 100, or 100 to 1000. The latter ratio is by far better because the *greatest sensitiveness of the bridge is obtained when the resistances of all of the branches, including the galvanometer resistance, are as nearly equal as possible.*

The following points relate to the use of any form of bridge. The galvanometer and battery circuits should each contain a key. The battery key should always be depressed before the galvanometer key. Very often these keys are mounted one under the other so that one motion closes both keys in the proper order. The galvanometer and battery can always be interchanged, but it can be shown that the best arrangement is that in which the galvanometer connects the junction of the two highest resistances to the junction of the two lowest. These conditions are usually fulfilled, in the case of the box bridge, when the battery leads connect to the junction of *a* and *b*, and the junction of *x* and *r*. One cell is sufficient for use with the bridge. The connections to the unknown resistance should be made as short as possible, and firmly made with heavy wire in order not to introduce any appreciable amount of extra resistance.

When using the box bridge, the plugs remaining in the bridge should be firmly inserted to minimize plug resistance. The plugs should be urged inward with a slight turning motion, but, on finishing with the bridge; the strain on the plugs and the cover should be relieved by loosening all plugs. It is not necessary to use great force in inserting the plugs. The unused plugs should be placed on a clean piece of paper to avoid the danger of dirt or mercury getting on the plug surfaces.

Under *no* conditions should a box bridge be connected to any source of electricity other than the one or two cells with which it is to be used. A current larger than 1/10 ampere should never pass through the resistance coils. In order to avoid this danger, even with the weak source of current used with the bridge, it is a good plan to unplug several ohms in the three parts of the bridge before closing the battery circuit.

Method. Connect up the slide-wire bridge, using a resistance box as the known resistance *r*. The double key should be so connected that the battery circuit is closed first. Measure the resistance of one of the unknowns. To correct, in part, for contact resistance at the ends of the meter wire, interchange the known and unknown resistances, and repeat the measurement. Take the mean of the two measurements.

Repeat the above measurements using another unknown resistance.

Measure with care the resistance of the same two unknowns in parallel, and also in series. Compare these results with the values for these combinations which are obtained by calculation from the separate resistances.

44. FAULT FINDING IN ARTIFICIAL LINES

Introductory Remarks. Electrical circuits very often become grounded or short-circuited at some unknown points hidden from view and, therefore, impossible to locate directly. This is true of circuits in electrical apparatus, of telephone and telegraph lines, of lighting circuits, etc. It is, therefore, very desirable to have some electrical means for determining the kind and position of the fault. The simplest and most convenient method of accomplishing this purpose is by resistance measurements at the two ends of the circuit. Certain common types of faults can be located in this manner, as will be made clear by the following experiment.

Object. The box form of Wheatstone bridge will be used in this experiments to illustrate the method of locating three common kinds of faults on lines. Exercise 43 contains a description of the box bridge.

Apparatus. Two boxes containing artificial lines each possessing a certain kind of fault, or combination of faults, are provided for test. Each line consists of two manganin wires of the same length, wound on a spool and enclosed in a box. The wires start at two binding posts at one end of the box and terminate in two binding posts, at the other end of the box. The ends of a single wire may or may not terminate in opposite binding posts. The total length of line, i. e., of one wire, is given on the box.

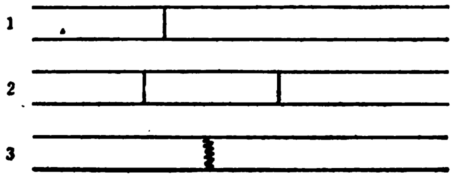

FIG. 30

The type of fault is not given, but the three types provided are shown in Fig. 30. The first represents a single short circuit of zero resistance, the second two such short circuits, and the third represents a connection between the lines which has resistance.

In addition to the above artificial lines, a box form of Wheatstone bridge, galvanometer, and cell are required.

Procedure. The method of obtaining the results should be carefully planned before beginning the experimental work. Measure all resistances necessary to completely determine the kind, the resistance, and the distance from the ends of the line of each fault.

Results. Calculate the position and kind of each fault, and show the method of obtaining the results.

It should be borne in mind that, in any actual case where the ends of the line are far separated, the resistance measurements would of necessity be made with no additional connection between ends. The actual cases would be solvable, however, provided the resistance of the ungrounded line were known.

45. SENSITIVENESS AND CALIBRATION OF A GALVANOMETER

Read the section on *Galvanometers and Ammeters*.

Introductory Remarks. Galvanometers are often used merely as current detectors. It is, however, of interest to know how sensitive the instrument is, i. e., to how small a current it will respond. The sensitiveness of reflecting instruments is usually stated as the current in amperes which will give a one millimeter deflection when the scale is one meter from the mirror. If the galvanometer possesses only a pointer, the sensitiveness is the current in amperes which will give a deflection of one scale division.

Galvanometers are very often used as ammeters to measure the strength of small currents. In this event it is necessary to calibrate the instrument, i. e., determine the currents which correspond to its readings. The results are usually expressed by means of a graph showing the deflections for various currents.

Object. The purpose of this experiment is to illustrate the manner in which a galvanometer is calibrated and its sensitiveness determined, and to give practice in the method of obtaining these results. The process will be applied to two galvanometers.

Apparatus. A small portable galvanometer and a sensitive reflecting instrument are provided for test and calibration. A slide-wire, consisting of a high resistance wire stretched along a meter rod and provided with a sliding contact, a Wheatstone bridge of the box form, and a cell of known voltage are provided.

Discussion. The only difficulty in calibrating a sensitive galvanometer is in obtaining a sufficiently small but known current. The method is based solely upon Ohm's Law.

Let G represent the resistance of the galvanometer, R the resistance unplugged from the Wheatstone bridge used as a plain resistance box, r the resistance per centimeter length of the slide-wire, and E the known voltage of the cell. If the connections are made as shown in Fig. 31, where *ab* represents the slide-wire, the current through the galvanometer can be made to have any value from zero to a current many times larger than is required for the experiment by changing the value of R and the position of the slider. The resistance R should be as large as possible in order that the current drawn from the cell may be small. If l is the distance in centimeters from a to the slider, then the current through the galvanometer is easily shown to be given by the following expression:

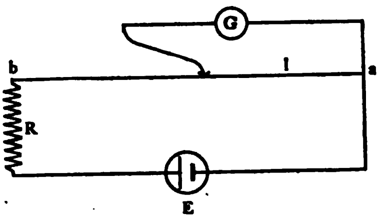

FIG. 31

$$I_G = \frac{Elr}{R\,(lr + G) + Glr}\,.$$

If the term lr can be shown to be negligible in comparison with G, the above expression reduces to

$$I_G = \frac{Elr}{G\,(R + lr)}\,.$$

Further, if lr is negligible as compared to R, the current through the galvanometer is given by the still simpler expression,

$$I_G = \frac{Elr}{GR}\,.$$

Procedure. Measure the resistance of the slide-wire by means of the Wheatstone bridge and portable galvanometer. It would be well not to use the standard cell for the bridge work.

The resistance of each galvanometer is next to be measured. This can be done in the usual manner by using the galvanometer not being measured for the bridge instrument. Care should be taken that too much current is not sent through the instruments.

Connect the apparatus as shown in Fig. 31, using first the portable galvanometer. Cause the instrument to read zero by adjusting the knurled head on the top of the box.

Record all necessary data for obtaining the complete calibration curve by the method outlined above. Each new adjustment should be made so that the galvanometer moves by just one division.

In the same manner obtain the data for the calibration curve of the reflecting d'Arsonval instrument. Record the distance of the scale from the mirror.

Results. Show the derivation of the formulae given in the discussion.

Calculate the current for each of the adjustments recorded, and plot the results for each instrument in a curve, for which the ordinates represent true current through the instrument, and the abscissae the corresponding readings of the instrument. One of the simpler formulae may be used if justifiable from point of view of accuracy.

From each curve, derive and state the sensitiveness of each instrument.

46. ELECTROLYSIS

Introductory Remarks. The practical unit of quantity of electricity is the *coulomb.* The *rate* at which electricity is transferred or flows along a conductor determines the strength of *current* which is measured in *amperes. An ampere is a current flowing at the rate of one coulomb per second.* If the current is perfectly steady, then the current I is equal to the quantity Q which flows in the interval of time t, divided by t. The total quantity of electricity transferred through a conductor is equal to the current times the time, *provided* the current remains constant.

An ammeter is an instrument for measuring the strength of a current of electricity, and if the current strength remains constant for a certain interval of time, then the ammeter reading multiplied by the time gives the number of coulombs which have flowed. In this way the ammeter can be used to measure quantity of electricity.

When a current of electricity passes through some aqueous solutions the latter are decomposed. This process is called *electrolysis*, and the solution thus decomposed is termed an *electrolyte*. The products of decomposition usually appear at the electrodes which conduct the current into and out of the solution. If the electrolyte is an acid, hydrogen appears at the *cathode* or the terminal by which the current leaves the solution. If the electrolyte is a solution of a metallic salt, the metal is often deposited on the cathode.

The amount of material liberated depends directly upon the quantity of charge transferred. The volume of hydrogen, under standard conditions of pressure and temperature, which is liberated by one coulomb of electricity, and the weight of copper and silver deposited under certain specific conditions by the unit quantity of electricity, have been very accurately determined. With these constants, the electrolysis cell or voltameter may be used to determine the strength of an unknown current. In the early days of electric lighting, electrolysis cells, known as coulombmeters, were installed in the houses to measure the consumption of electricity. The most important application of the voltameter is its use in calibrating an ammeter or other current measuring device.

Object. The object of this experiment is to determine the electro-chemical equivalent of copper and of hydrogen.

Apparatus. The *gas voltameter* consists of two vertical tubes graduated in cubic centimeters, closed with stopcocks at their upper ends, and joined together below. A platinum electrode is sealed into each tube at its lower part. The apparatus is filled with dilute sulphuric acid by means of a standpipe.

The *copper voltameter*, as used in this experiment, consists of a thin sheet of copper, 4 x 6 cms. in dimensions, hung on the spring of a Joly balance between two copper anodes. Means are provided for making an electrical connection with the suspended plate, which is to be the cathode or gain plate.

Current is derived from a set of storage cells. An adjustable resistance for regulating the current and a one-ampere ammeter are provided.

Discussion. The weight in grams of substance deposited or set free by a quantity of electricity Q coulombs, or by the constant current of I amperes flowing for the time t seconds, is

$$W = K\,Q = K\,I\,t,$$

where K is the electro-chemical equivalent, or the number of grams of substance electrolyzed by one coulomb. If one of the products of electrolysis is a gas, as hydrogen for instance, it is much easier to measure the amount liberated in terms of its volume. The above formula may then be stated

$$V = K'Q = K'It$$

where V is the number of cubic centimeters of gas set free at 76 cms. pressure and 0 C., and K' is the volume of gas set free under standard conditions by one coulomb.

For accurate results with the hydrogen voltameter the electrolyte should be a chemically pure 10% solution of sulphuric acid. Distilled water should be used in mixing the solution.

The copper voltameter should be arranged to have a certain concentration of electrolyte, temperature, and current density per square centimeter of cathode surface. The electrolyte should be a one half saturated solution of copper sulphate to which 1 % of sulphuric acid has been added. The proper saturation is given by dissolving 200 grams of copper sulphate in 1000 c.c. of water. The function of the acid is to hold in solution iron and other metal impurities. The temperature should be about 20° C., although a temperature a few degrees higher or lower will cause no serious error. The cathode should have an area of about 50 square centimeters for every ampere of current.

The following special directions must be followed with the particular apparatus used in this experiment.

The volume of gas liberated in the gas voltameter, and obtained by reading the level of the liquid in the graduated tube, must be reduced to the volume which the gas would occupy if the pressure were 76 cms. of mercury and the temperature 0°C. The pressure in the tube, obtained by correcting the barometer reading for the height of the acid in the standpipe above the acid in the tube, is not the pressure of the gas alone, but the *sum* of the pressures of the hydrogen and the water vapor. The pressure of water vapor which must be subtracted from the total pressure to give the pressure of the gas alone, depends upon the strength of acid solution and the temperature. Table 3 in the *Appendix* gives this correction for different conditions.

The apparent gain in weight of the cathode of the copper voltameter, obtained from the readings of the Joly balance taken while the cathode is submerged, is not the true gain in weight because of the buoyancy of the copper sulphate solution on the deposited copper. If W is the true weight of deposited copper and W' the apparent gain in weight of the cathode, then

$$W = \frac{W'}{1 - \frac{d}{D}},$$

where d is the density of the copper sulphate solution, and D is the density of copper.

Procedure. (1) Fill the hydrogen voltameter so that the level of the acid in the standpipe is a centimeter or two above the ends of the stop-cocks. Connect a one-ampere ammeter in series with the gas voltameter and the variable resistance, and connect to the switch which supplies the current. At an exactly determined instant, close the switch, and, by means of the regulating resistance, maintain the current absolutely constant at about .25 ampere. As soon as the hydrogen has filled the tube to about 40 c.c., open the circuit, noting the exact instant. Record the reading of the meniscus in the hydrogen and oxygen tubes, and the height of the liquid in the standpipe above the level of the liquid in each graduated tube. Make a note of the room temperature and the barometric pressure.

(2) Adjust and level the Joly balance and copper cell so that the suspension system hangs freely and the gain plate is midway between the two copper anodes and entirely submerged in the copper sulphate solution. Only a single fine wire should extend through the surface of the liquid.

Make connection to the gain plate by the means provided, and connect the ammeter and resistance in circuit. Make sure that the gain plate is the cathode. Close the switch just long enough to adjust the current to .8 or .9 ampere. Remove the connection from the gain plate, and accurately read the Joly balance. Once more make connection to the gain plate, and, at an exactly noted instant, close the switch. Maintain the current constant for at least three quarters of an hour. Note the exact instant the switch is opened. After disconnecting the gain plate, read the Joly balance.

The spring of the Joly balance must now be calibrated by noting the stretch produced by a one gram weight.

The density of the copper sulphate is now to be determined by weighing, by means of a second Joly balance, the small glass sinker in the copper sulphate and in water; or its value may be furnished by the instructor.

Results. Calculate the electro-chemical equivalent for the two cases. The density of the acid solution used in the gas voltameter may be assumed to be 1.18 gms. per c.c. The density of copper is 8.9 gms. per c.c.

47. POTENTIOMETER

Introductory Remarks. The measurement of voltages or electromotive forces is of evident necessity in practical and experimental work. This is often accomplished by means of voltmeters, but these instruments are simply galvanometers which were first calibrated, and are often recalibrated by means of the much more accurate potential measuring device, the *potentiometer*.

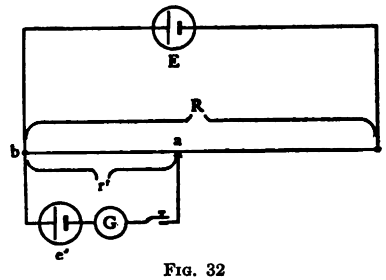

FIG. 32

THE POTENTIOMETER

The potentiometer is an apparatus for very accurately determining an unknown voltage in terms of the voltage of a standard cell, which has been very accurately measured and adopted as a legal standard. The potentiometer has the great advantage over many other methods of measuring potential in that *no* current is drawn from the source of potential being measured. On the other hand, almost all voltmeters require some current to give the deflection of the instrument, and this current, although small, when derived from the potential being measured, often so changes conditions that the voltage measured is not the voltage which exists before the instrument is connected.

The principle of the potentiometer is embodied in Ohm's law, i. e., voltage = current x resistance $\left(I = \frac{E}{R}\right)$. In its simplest form, the potentiometer consists of a high resistance R through which a few cells cause a small constant current i to flow (see Fig. 32). A standard source of potential e' is then connected, as in the figure, through a key and galvanometer and across some of the resistance R. The direction of e' must be so as to tend to cause a current to flow through the resistance r' in the same direction as the current caused by the main set of cells. If the point of contact a be moved, a point will be found, provided e' is less than the electromotive force of the main battery, when no current will flow through the galvanometer because the potential drop through the portion of the resistance between the terminals of the side circuit is equal and opposite to e'. Under these conditions, if r' is the resistance bridged across by the circuit containing e', then $e' = ir'$. If now e' be replaced by an unknown electromotive force E', and a new point of balance be found, so that R' is the resistance bridged across, then $E' = iR'$. Combining the two expressions above, we have

$$\frac{E'}{e'} = \frac{R'}{r'} \text{ or } E' = e' \frac{R'}{r'}$$

If the galvanometer is sufficiently sensitive, the values of R' and r' can easily be determined to 4 or 5 significant figures, which gives a measure of E' accurate to .01% or better.

Potentiometers are made up in very convenient forms, but excellent results can be obtained with the very simple apparatus, to be used in the following experiment.

Object. The object of this experiment is to give experience in the use of a very simple potentiometer as applied to the measurement of the voltage at the terminals of a Daniell's cell, as a function of the current drawn from the cell. Incidentally, the results will give some interesting information concerning the properties and action of the Daniell's cell.

Apparatus. The potentiometer which is to be used consists of a pair of resistance boxes, connected in series, which take the place of the parts of the slide wire on each side of the point a in the figure, which is the point between the two boxes. For simplicity, it is necessary to keep the *sum* of the resistances in the two boxes always equal to 1,000 ohms. If this is done, the action of the apparatus is similar to that of the slide wire. The accessory apparatus consists of a portable galvanometer, a key, a dry cell to be used as the standard cell, a Daniell's cell, and a resistance box, the use of which will be explained later. The current for the potentiometer is obtained from a storage cell.

Discussion. If a current I be drawn from a cell which has an electromotive force E, and an internal resistance r, the difference of potential at the terminals of the cell is given by the expression

$$V = E - Ir$$

This means that the external voltage of the cell is less than the true electromotive force by the amount of the fall of potential in the cell itself. If the current is supplied to an external resistance R, the current is given by the relation

$$I = \frac{V}{R}.$$

If the external resistance R be now varied, I and V will change. The variation of V with respect to I can be represented by a graph, for which the ordinates are the values of V, and the abscissae the corresponding values of I. The first expression given above is the equation of the graph. It is clear that, if E and r remain constant, the graph will be a straight line which cuts the axis of ordinates at the value E, and the axis of abscissae at the value $\frac{E}{r}$. The slope of the line is the value of r, since the slope is the ratio of E to $\frac{E}{r}$. If E and r do not remain constant, the graph will in general be curved, and an examination of the line will give some idea of the way in which E or r changes.

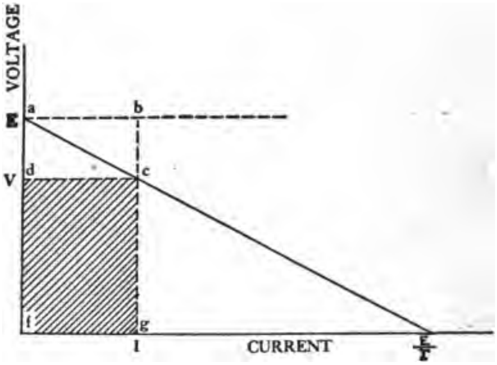

FIG. 33

In addition to the information concerning the constancy of E and r, the graph gives some very interesting data concerning the distribution of power in the circuit and the efficiency of the cell under various conditions. Let Fig. 33 represent the straight line graph for a cell in which both E and r remain constant. Let cg be the ordinate corresponding to any current I. It will be clear, then, that the area of the rectangle $abgf$ represents the total power being produced by the chemical action. The shaded area gives the power dissipated in the external resistance, and the area of the rectangle $abcd$ represents the power dissipated in heat in the internal resistance of the cell. The efficiency of the cell for the current I is, therefore, $\frac{\text{area of } dcgf}{\text{area of } abgf}$. It can easily be shown that the shaded area will be a maximum for a current equal to $\frac{E}{2r}$, *one half* the maximum current of the cell. The graph, therefore, shows the power output and efficiency for any current; and, in addition, the current, voltage, and efficiency, for maximum power output of the cell. The above discussion holds with some limitations to a curved graph.

Procedure. Connect the two large resistances boxes, the storage battery and the upper half of the double key together, all in series, setting the sum of the resistances in the two boxes to the value 1000 ohms before connecting them in. Connect the standard dry cell, the galvanometer and the lower half of the double key, after the manner indicated in Fig 32 (in series with one another) to the ends of *one* of the large resistance boxes. The resistance of this box will be the resistance r' of the figure, while the resistance R of the figure is 1000 ohms. Record the voltage of the standard cell, and the reading for the point of balance. Large currents should never be drawn from the standard cell.

Replace the standard cell by a Daniell's cell, and record the new point of balance.

Connect the third resistance box across the terminals of the Daniell cell, adjusting its plugs so that its resistance is 20 ohms (this connection is not shown in Fig. 32). Arrange the cell and wires so that the position of the zinc and copper parts (the "elements") of the cell will not in any way be disturbed. This precaution is necessary in order that the internal resistance of the cell will not vary because of a change in distance between the elements. Record the point of balance in each case when 20, 12, 8, 4, 2, and 1 ohms are separately unplugged in the box.

Results. Calculate the voltage of the Daniell cell as it was at first, and again after the resistance box was connected to it, for each value of the resistance in this box.

Plot the curve described in the discussion for the Daniell cell. State the open circuit potential and internal resistance; the latter is found from the value of $\frac{E}{r}$ determined from the graph.

From the plot for the Daniell's cell, determine and state the maximum power output, and the current, voltage, external resistance, and efficiency for the condition of maximum power output.

48. MECHANICAL EQUIVALENT OF HEAT BY ELECTRICAL METHOD

Introductory Remarks. The electrical units are so defined that the relation between them and the mechanical units is definitely known. For instance, the work done by a current is, by definition, equal to the product of the three factors, the current in amperes, the potential difference, in volts and the time in seconds. The answer comes out in Joules, where one Joule is simply a multiple of the erg, i. e., ten million ergs. Definition does not establish the relation between the electrical units and the heat units any more than does definition establish the relation between the mechanical units and the heat units.

The very important constant, the mechanical equivalent of heat, can be determined by measuring the quantity of heat produced directly by a known amount of mechanical work as was done in an earlier experiment, or by the much more accurate method of determining the electric equivalent of heat by finding the quantity of heat produced by the expenditure of a measured amount of electrical energy, and then converting this result into the mechanical equivalent of heat by means of the known relation between the electrical and mechanical units.

The latter process will be the method used in this experiment and consists essentially of measuring, by means of a calorimeter, the quantity of heat produced in a given time by a measured current of electricity flowing through a heating coil of known resistance placed inside the calorimeter.

The measurement of the heat will be accomplished by the so-called *continuous flow calorimeter*, a type very often used in practice.

The principle upon which this type of calorimeter works is as follows: A constant stream of water flows through the apparatus which contains the electrical heating coil, and the rate at which the water flows is determined by weighing the amount of water used in a certain time. The temperatures of the ingoing and the outgoing water are accurately measured by means of two sensitive thermometers. If the apparatus is so constructed that there is no loss of heat by radiation or conduction from the water as it passes between the two thermometers, and if the water be allowed to flow at a perfectly constant rate *until* the difference in temperatures, as recorded by the two thermometers, remains *constant*, then it is easily seen that all of the heat produced by the current of electricity goes to heating the water from the temperature of the inlet to the temperature of the outlet. Corrections for heat absorbed by the material of the apparatus or for heat lost are evidently unnecessary.

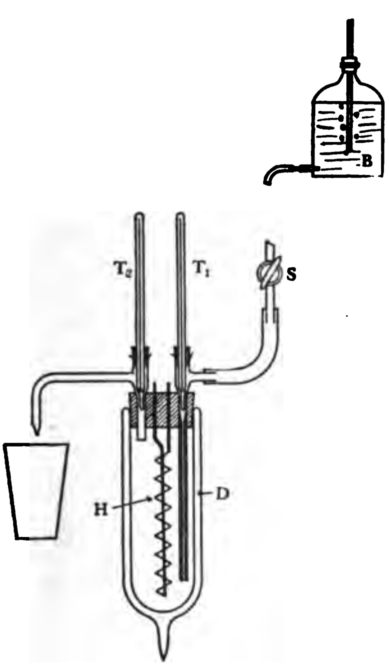

Fig. 34

Object. The object of this experiment is to give experience in the use of the continuous flow calorimeter as applied to the determination of the mechanical equivalent of heat according to the method outlined above.

Apparatus. The construction of the continuous flow calorimeter, which will be used in this experiment, is shown in Fig. 34. The vacuum or Dewar flask *D* consists of a double walled tube, the opposing surfaces of which are silvered to diminish transfer of heat by radiation. The chamber between the walls is highly exhausted to prevent transfer of heat by conduction or convection. This arrangement very efficiently insulates the interior cavity from loss or gain of heat through the walls. *H* is an electric heating coil made of manganin resistance wire, an alloy which changes very little in resistance with change in temperature. Water flows from a bottle B, so arranged that the rate of flow is maintained constant, through the regulating stop-cock S, past the inlet thermometer T_1, and into the bottom of the calorimeter through a thick small-bore glass tube. The velocity of the water through the small bore of the tube is considerable, and there is little probability of heat flowing up the stream of water and affecting the thermometer T_1. The water is deflected as it goes about the heating coil by mica vanes or partitions not shown in the figure, and then flows out past the outlet thermometer T_2.

The heating coil H is connected in series with a 5-ampere ammeter and a regulating resistance. Current is supplied from a set of storage cells. A voltmeter is supplied for measuring the fall of potential through the resistance.

Procedure. Turn the stop-cock part way so that the water flows from the calorimeter in a small but continuous stream. As soon as the readings of the thermometers become *constant*, record the temperatures every minute for 5 or 6 minutes. The two thermometers may differ because of errors in their absolute calibration. Since the *length* of a degree is, however, probably correct, the discrepancy in the readings will not affect the results. Now turn on the current, and adjust it so that there is an opportunity both to decrease and increase the variable resistance for regulation. It is the duty of one man to maintain the current absolutely constant, for, since the current is squared to give the heat produced, a slight change in the strength of the current will have a large effect upon the result. The difference in temperatures should be over 5° and not over 15°.

Weigh one of the metal cans. As soon as the temperatures become constant, note the exact time and place the weighed can under the outlet. Record the temperatures every 30 seconds until the can is about three quarters full. Note the exact time when the can is removed. Weigh the collected water.

Change the rate of flow of water and repeat the run, exchanging duties. At least two independent runs should be made.

Find the resistance of the heating coil which is obtained from the voltmeter and ammeter readings.

Note. Unless certain precautions are observed the results of this experiment are likely to be in error. The bulbs of the two thermometers should project as far into the small tubes as possible and yet allow the water to flow. The large tubes into which the stoppers that hold the thermometers fit and the outlet spout should be completely full of water. This is necessary in order that the rate of flow may not change. The water in the bottle *B* should be at room temperature.

Results. The result of each run is to be calculated and expressed in joules per calorie.

In working up the results, the average of the inlet thermometer readings is to be subtracted from the average of the outlet thermometer readings. The result thus obtained is the average rise of temperature of the water.

49. TEMPERATURE MEASUREMENT BY THERMO-COUPLE. LATENT HEAT OF FUSION AND MELTING POINT OF TIN

Introductory Remarks. The two junctions formed by connecting a wire of one metal in a circuit consisting of another metal are sources of oppositely directed electromotive forces, the magnitudes of which depend upon the temperatures of the junctions. If the junctions are at the same temperature, the electromotive forces are equal and opposite, and, therefore, neutralize each other so far as producing a current in the circuit is concerned. If, however, one junction is at a higher temperature than the other, there will be a resultant electromotive force in the circuit which can be used to measure the difference in temperature of the junctions. Such junctions are called *thermo-couples*, and are very commonly used as thermometers.

The usual practice in thermo-electric thermometry is to maintain one junction at 0°C., and to subject the other to the temperature being measured. The resulting electromotive force is then measured by means of a potentiometer, as in this experiment. If the junction has already been calibrated in terms of temperature, or if the law connecting temperature and electromotive force for the junction is known, the unknown temperature can be obtained.

Various combinations of metals are used. For very high temperature measurements, the refractory metals, platinum and an alloy of platinum and iridium, or of platinum and rhodium, are the best. For lower temperatures, copper with constantan, an alloy of copper and nickel, gives very good results because of the high electromotive force produced, and because of the practically linear relation between the electromotive force and the temperature of the hot junction, if the latter temperature be not too high. Chromel and alumel are two alloys of recent discovery which form a combination giving a still more nearly linear curve.

Thermo-couples are usually calibrated by subjecting the hot junction to several standard temperatures such as the boiling point of water, the melting points of tin, lead, etc.

Object. The object of this experiment is to illustrate the method of calibrating and using a thermo-couple for the measurement of temperature. The temperature of some tin will be traced as it freezes. The temperature of steam will also be tested. From data obtained from the observations, the melting point and latent heat of fusion of the tin will be determined.

Apparatus. The thermo-couples used in this experiment are made from the two alloys chromel and alumel. They are so mounted that the junctions themselves are at the bottom of little glass tubes, whereby they may be handled. The wire connecting these tubes is of one alloy; the two end wires of the other. A jar is needed, full of cracked ice, with very little water, in which one of the junctions is to be immersed throughout the experiment. The other junction is to be put in succession into the top of the steam boiler, or well down into the bath of tin. This bath is a crucible, so mounted that it is easy to heat it. Be careful not to heat it much above the melting point of the metal.

Method. A potentiometer arrangement is to be used in this experiment which is quite similar to that shown in Fig. 32, page 85. The thermo-couple takes the place of the cell *e*, and the same precautions must be taken to see that it is connected in to the circuit in the proper direction. A dry cell furnishes a small but steady current through the slide wire, and also through an adjustable resistance, not shown in the figure. The difference of potential produced by the couple is to be balanced against the fall of potential through part of the slide wire. A double key should be used to keep both parts of the circuit open when not in use. The galvanometer is of the type referred to on page 73.

It would be convenient if divisions on the meter bar of the slide wire corresponded in some simple way to degrees Centigrade, at least approximately. In order to arrange this, put the hot junction in the steam in the boiling apparatus, and the cold junction in ice. Then adjust the current through the slide wire by means of the resistance until there is no current through the galvanometer when the slide wire key is at 20 cm. (that is, $a\ b$ = 20 cm. in Fig. 32). Then 2 mm. on the scale = 1°C; and temperatures may be read directly from the scale with fair precision. This assumes that the voltage of the couple is proportional to temperature, which is approximately true.

Then proceed as follows: — light the burner under the tin and melt it. With the hot junction in the tin, the point of balance will move over toward, or beyond the middle of the scale. Run the temperature up to 300° or so; then turn off the heat, and begin to record the point of balance on the scale (i. e., the temperature) and the time about every 30 seconds, continuing until the temperature has fallen to about 100°C. In finding the point of balance it is convenient to proceed as follows: — note the direction of motion of the galvanometer reading which corresponds to cooling, and call this, for convenience, *down*. To get a point of balance, force the galvanometer *up*, a little past the zero (or point of rest) by moving the slider on the bridge; then wait, holding the slider still and the keys down, until the galvanometer crosses zero. Note the exact time when this happens; note also the slider position, and as soon as these two are recorded, repeat the process. Beware, however, of the possibility that at the melting point the galvanometer will not go down; if this happens, move the slider at once to the position which brings the galvanometer to zero, and note time, as before, every 30 seconds, and until the galvanometer indicates a resumption of the cooling process; after this follow it, as before.

Results. Plot these observations on a sheet showing temperatures one way, and time the other. From the level part of the curve, determine the observed melting point for the metal.

From the curve for tin, find the tangent to this curve, in degrees per minute, just before solidification begins. Find also the tangent to the curve when cooling begins again, after solidification; there will be a bend in the curve there, but the tangent is to be drawn to that part of the curve where it becomes approximately straight. These two tangents will be nearly parallel, and would be more closely so if the specific heats of the liquid and solid states were equal. The distance *parallel to the time axis* between these lines gives the time during which solidification occurs. The number of calories lost per minute before solidification is equal to the rate of cooling in degrees per minute (t) multiplied by the thermal (or "water") equivalent of the tin and its crucible. If M is the mass of the tin and S its specific heat (0.054) while m and s are corresponding quantities for the crucible, the number of calories lost per minute before solidification becomes

$$(MS + ms)\ t.$$

During solidification the crucible gives out no heat of its own, but the tin gives off latent heat. L calories per gram. If it takes T minutes to freeze, there must be LM/T calories given off by the tin per minute during this process. But the rate at which heat is given off is approximately constant all this time, hence

$$LM/T = (MS + ms)\ t.$$

If the crucible is so light and thin that it can be neglected this reduces to $L = tTS$. But usually this is not the case, and we get

$$L = tT\left(S + \frac{ms}{M}\right).$$

Use the last equation for finding L. Weigh the tin in its crucible; the weight of the crucible alone is given; so is its specific heat.

50. THE GENERATOR

Object. The object of the experiment is to determine some of the principal characteristics of a shunt-wound D. C. Generator.

Method. Examine the motor-generator set to determine the rated voltage and power of each machine, and observe that the generator connections are as indicated in fig. 35.

Arm is the armature with brushes b_1, b_2 which are connected to the terminals T_1 T_2.

V_1 is a voltmeter connected to the terminals T_1, T_2 giving the voltage supplied to those terminals by the armature. The reading of V_1 is called the terminal voltage and will be designated as E.

F is the field-magnet coil.

R is a variable resistance connected in series with F.

R_L is a variable resistance through which the external (or *load*) current supplied by the generator is driven. It is connected to the terminals T_1 T_2.

A_L and A_F are ammeters in the external (or load) and field circuits respectively. We will call their readings I_l and I_f

A tachometer is provided for determining n, the r.p.m. of the generator. n may be varied at will by means of a variable resistance in the armature circuit of the motor. There are also instruments for determining the current I_m and voltage E_m supplied to the motor. (Tachometer and motor connections not shown in fig. 35).

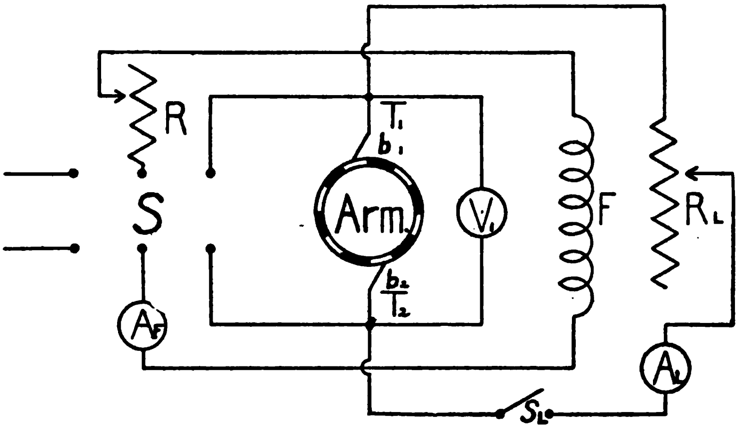

FIG. 35

An efficiency curve of the motor is posted in the laboratory.

Case 1. To determine the relation between I_f and E' when n is constant:

The voltage generated in an armature is

$$E' = Kn\phi \qquad (a)$$

where n is the number of revolutions per minute, ϕ the number of lines of force which the field magnets cause to pass through the armature, and K a constant depending upon the manner in which the armature is wound. If n is kept constant E' is directly proportional to ϕ, and a graph of E' plotted against I_f will be a magnetization curve for the field magnets. Note that the voltmeter reading E is the same as E' if no current is being drawn from the armature.

Throw the switch S to the left thus connecting the field circuit across the 110 volt mains. The field magnets are now in no way connected to the armature. The field circuit current may be varied at will by means of the variable resistance R. See that switch S_L is open, thus preventing the passage of current from the armature through R_L.

Maintain n at 1100 r.p.m. by adjusting a resistance R_m not shown in the diagram. Vary I_f in steps of one tenth ampere from zero to one half ampere, and in steps of two tenths ampere from one half to two and a half amperes, recording the terminal voltage E, as well as I_f, in each case. The current, I_f must always be increased from one value to the next and never decreased. If it is decreased the observed point may, on account of hysteresis, not lie on the magnetization curve.

Plot E' as ordinates against I_f as abscissae.

Case 2. To determine the relation between E' and n when ϕ is constant:

Keep the switch S_L open as in Case 1. With S again thrown to the left, ϕ is to be kept constant by maintaining I_f at some constant value such as half an ampere. Vary n in steps of fifty r.p.m. through as wide a range as possible.

Plot E' as ordinates against n as abscissae. The curve thus obtained should be a straight line and is a check on the formula $E' = Kn\phi$. The slope of the line is equal to the product $K\phi$. What sort of curve would have been obtained if I_f had been one instead of one half ampere?

Case 3. To determine the relation between terminal voltage, output, efficiency, and the external (or load) current:

Throw the switch S to the right, thus connecting the field windings to the armature terminals T_1 T_2, as is always the case in normal operation. Make the resistance $R = 0$. Maintain n at 1100 r.p.m. Close the switch S_L. The load resistance R_L is to be varied from maximum to zero in steps of such size that the current I_l will vary in steps of about one ampere. Record I_f, and E of the generator and I_m and E_m of the motor for each value of I_l. I_l is, of course, also to be recorded in each case. The efficiency of the motor for each value of $E_m I_m$ is to be obtained from the motor efficiency curve.

Note that the terminal voltage E is no longer equal to the generated voltage E' but is

$$E = E' - I_a R_a \qquad (b)$$

where $I_a R_a$ is the voltage drop in the armature due to the passage of the current I_a through the resistance R_a of the armature itself.

Since the armature supplies both field and external circuits

$$I_a = I_f + I_l \qquad (c)$$

From equation *c* it is seen that the first increase of I_l must cause an increase in I_a. The increase in I_a causes E to become smaller (equation *b*). I_f must then become smaller because

$$I_f = \frac{E}{\text{Resistance of Field Circuit}} \qquad (d)$$

When I_f becomes smaller ϕ becomes smaller according to the relation expressed in the curve obtained from Case 1. As ϕ decreases so also does E'. The decrease of E' further decreases E, but the final value of E is such that

$$I_l = \frac{E}{R_L} \qquad (e)$$

Note that the decrease of E makes the external current smaller for any given value of load resistance than it would otherwise have been. (Equation *e*.) With each decrease of R_L the process of lowering E and E' is repeated. (This lowering is also increased in each case by so-called "armature reaction," which is the distortion and effective decrease of ϕ by the opposing action of other lines of force produced by the current in the armature itself). When I_l has been increased until I_a is 12 or 15 amperes the term $I_a R_a$ (in equation *b*) is so large compared with E', and the armature reaction becomes so large, that a further decrease of external resistance causes I_l to become smaller instead of larger. (See equation *e* and remember that while R_L is decreased, E decreases at a faster rate.)

When R_L is decreased to zero, $I_a R_a$ becomes equal to E', and E is then zero.

The following curves are to be plotted on one sheet of graph paper and the same scale of abscissae used for all of them:

Observed values of E as ordinates against observed values of I_l as abscissae. This curve is called the "external characteristic" of the generator.

Calculated output EI_l as ordinates against observed I_l as abscissae.

Calculated values of generator efficiency, as obtained from

$$\text{Efficiency} = \frac{EI_l}{E_m I_m \times \text{(appropriate efficiency of motor)}}$$

as ordinates against observed I_l as abscissae.

From the three curves find:

The terminal voltage, output, and efficiency for maximum load current.

The terminal voltage, load current, and efficiency for maximum output.

The terminal voltage, load current, and output for maximum efficiency.

51. THE MOTOR

Object. The object of the experiment is to determine some of the characteristics of a shunt wound D.C. motor.

Method. The electrical connections are indicated in Fig. 36. *Arm.* is the armature with brushes b_1 b_2 which are connected to the terminals T_1 and T_2.

F is the field-magnet coil.

B is the starting box.

r' is the coil working the "release" lever L of the starting box.

A_A and A_F are ammeters for determining the currents I_a and I_f of the armature and field circuits respectively.

V is a voltmeter giving the voltage between the terminals T_1 and T_2. This is the "terminal voltage" and will be designated as E.

A tachometer is provided for determining n, the number of r.p.m. of the motor.

S is the switch for connecting the motor to the supply current mains.

A prony brake like that shown in Fig. 19 is provided for use in determining the brake output of the motor. B is for the purpose of exactly balancing the long arm of the brake so that at rest the balance reads zero. By means of the screw S the bar of the brake may be adjusted so as to remain horizontal for all loads. The two halves are forced together and the motor loaded by means of the two nuts NN, each of which should be screwed down by the same amount in order to keep the parts of the brake in line.

In order to make the armature of a motor rotate, a current must be driven through the armature by means of an electromotive force, E, applied to the armature through the brushes. As the armature rotates the armature conductors cut the lines of force created by the field magnets and this cutting of the lines produces an electromotive force in the armature conductors. This electromotive force is, by Lenz's law, in the opposite direction to the electromotive force E. It is called the "back e.m.f." and is denoted by E_b. E_b is given by the equation

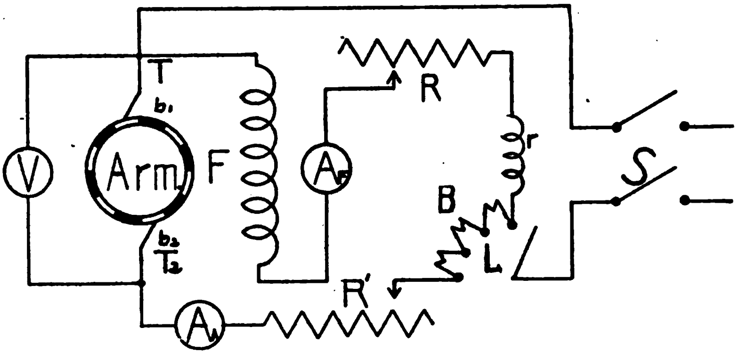

Fig. 36

$$E_b = Kn\phi \qquad (a)$$

where, ϕ is the number of lines of force which the field magnets cause to pass through the armature, n is the number of r.p.m., and K is a constant depending upon the manner in which the armature is wound. Since E_b opposes the action of E, the armature current will be determined by the difference between E and E_b. It is given by the equation

$$I_a = \frac{E - E_b}{R_a} \qquad (b)$$

where I_a is the current passing through the armature and R_a is the armature resistance.

A study of equations a and b enables us to predict the behavior of the motor under any circumstances whatever. Equation b may be put in the form,

$$E = E_b + I_aR_a \qquad (c)$$

I_aR_a is voltage drop in the armature. The voltmeter connected across the armature gives E. Let us multiply each side of equation (c) by I_a, and obtain,

$$EI_a = E_bI_a + I_a^2R \qquad (d)$$

If to each side of this equation we add EI_f, we obtain

$$E(I_a + I_f) = E_bI_a + I_a^2R + EI_f \qquad (e)$$

$I_f + I_a$ is the total current used by the motor and $E(I_a + I_f)$ is consequently the "input," or total power used by the motor. On the right hand side of equation c we have a term I_a^2R which is obviously power lost in the form of heat produced in the armature. EI_f is power used in the field magnet coils. E_bI_a, the only other term in the right hand side of the equation, must then be the power used in producing the rotation of the armature. Let us call this power the "indicated output," it being the output as indicated by the equation. (It is analogous to the output of a steam engine as given by an "indicator" diagram). The "indicated efficiency" is then,

$$\text{"indicated efficiency"} = \frac{E_bI_a}{E(I_f + I_a)} \qquad (f)$$

Three sets of observations are to be taken.

Case 1. To determine how E_b and n vary with the voltage applied to the armature:

Make the resistance R equal to zero and make R' as large as possible. Close the switch S and start the motor. Since the resistance R' is in series with the armature there is a voltage drop I_aR' in it, and the voltage applied to the armature terminals is less than the line voltage 110 by the amount I_aR'. Record the terminal voltage E (from the reading of V) and speed n for several values of R' from its maximum to zero. Record the resistance R_a of the armature itself. This is posted near the machine. Using equation c calculate E_b for each observation.

Plot E_b as ordinates against n as abscissae. Show that the resulting curve is consistent with what would be expected from equation a.

Case 2. To determine how n varies with I_f and how E_b behaves during these variations.

With resistance R' equal to zero, vary R in such manner as to decrease I_f in steps of one hundredth of an ampere from its maximum value (about .2 ampere) to one tenth ampere. Record I_f, I_a, E and n.

Plot I_f as ordinates against n as abscissae. Notice that E_b as given by equation c remains constant. How can this be consistent with equation a?

Case 3. To obtain the efficiency curves of the motor under normal operating conditions:

Make R and R' each equal to zero. Put the prony brake in place as shown in Fig. 19. Close S and start the motor. Tighten the nuts NN so as to vary the load in such steps that the readings of the balance vary, in steps of one quarter pound, from zero to about two and a half pounds. Record for each load E, I_a, I_f, n, and the reading of the balance. Use the equation f to calculate the "indicated efficiency" for each load.

Plot "indicated efficiency" as ordinates against watts output as abscissae. Calculate the brake output from "Output" = $2\pi nrF$ foot-pounds per minute, where F is the balance reading, n the r.p.m., and r the length of the brake lever arm. (See Fig. 19.) Each of these values of output may be changed to watts by multiplying $2\pi\, n\, r\, F$ by $\frac{746}{33000}$.

The brake efficiency is given by

$$\text{Brake efficiency} = \frac{2\pi\, n\, r\, F \times 746}{33000 \times E(I_a + I_f)} = 0.142 \times \frac{nrF}{E(I_a + I_f)}.$$

Plot brake efficiency as ordinates against output in watts as abscissae. Plot this curve on the same graph sheet with the "indicated efficiency" curve. Explain why one efficiency is always less than the other.

52. A STUDY OF THE PROPERTIES OF IRON IN A MAGNETIC FIELD

Introductory Remarks. The great majority of electrical machines and devices depend in their action upon the magnetic effects of an electric current. In these devices, strong magnetic effects are usually obtained by the use of many turns of wire, and by the use of iron as a part, or as the whole of the magnetic circuit. A knowledge of the behavior of iron in a magnetic field is, therefore, of the greatest interest and importance.

Object. The object of the following experiment is to give some elementary knowledge of the properties of iron when subjected to magnetic fields of various strengths. The magnetic strength of a piece of soft steel and a piece of hardened steel will be traced as the specimen is subjected to a first increasing and then decreasing magnetizing force.

Apparatus. The instrument used for the comparative measurement of the magnetic strength of the specimens is known as a *magnetometer*. This instrument consists of a small suspended magnet and mirror, the small deflections of which can be read by a telescope and scale. A horizontal meter rod, attached to the framework of the instrument at the height of the needle, serves as a support for the specimen. If the meter rod points in an east and west direction, and the short specimen is placed on the rod so that it is some distance from the needle and its axis passes through the center of the needle, the tangent of the angle through which the needle turns can be shown to be proportional to the *magnetic moment* of the specimen. The *magnetic moment* of a bar magnet is defined as the product of the strength of pole and its length, and is, therefore, a measure of the strength of magnetization. If the magnetometer is read by means of a mirror and scale, it should be noted that the scale deflection of the instrument divided by the distance from the mirror to the scale is the tangent of *twice* the angle of deflection.

A coil of wire about two inches long and wound with about 1100 turns of wire, is mounted at one end of the magnetometer meter rod, and is used for producing the magnetizing fields for the specimens.

The two specimens to be tested consist of a soft steel bar and a similar glass-hard bar. These bars are about two inches long by a quarter inch in diameter, and fit in a hole in the exciting coil.

An ammeter for measuring the current, and an adjustable resistance for varying the current in the coil, are provided. Both the ammeter and the resistance must be located at a considerable distance from the magnetometer because of their disturbing magnetic fields.

Current is taken from a set of storage cells.

Procedure. 1. Readings of the magnetometer and ammeter are to be taken, using the coil alone, as the current is increased from zero to the maximum in steps of about .1 ampere.

2. Place the piece of soft iron centrally inside of the coil, and increase the current by small steps from zero to the maximum and back again, taking all the necessary readings. The increase and decrease of the current must be *continuous*, that is, never in increasing the current must the current be decreased by any small amount, and similarly, never in decreasing the current must the current be increased by any small amount. This point is exceedingly important.

3. Repeat the same process with the hard steel bar. Before this bar is used, however, it must be demagnetized by withdrawing slowly from an alternating magnetic field. See the instructor.

If time permits, again increase the current to some value less than the maximum value and then return to zero current, taking the necessary readings in small steps.

Record the distance from the mirror of the magnetometer to the scale.

Zero readings of the instrument should be occasionally taken throughout the work.

Computation and Results. If θ is the angle of deflection of the magnetometer when acted upon by a specimen of strength M (magnetic moment) then

$$\tan \theta = KM, \text{ where } K \text{ is a constant.}$$

The deflection, as read on the scale, divided by the scale distance, is the tangent of $2\,\theta$. That is

$$\tan 2\,\theta = \frac{D}{L}\,.$$

In order to obtain data from the magnetometer which are *proportional* to the values of M, it is necessary to have an expression between M and D, or since $\tan \theta$ is proportional to M, between $\tan \theta$ and D. This can best be expressed graphically. Divide several specific values of D, as, for instance, multiples of 5 from 0 to 40, by the scale distance L. The results are the values of $\tan 2\,\theta$ from which the values of $\tan \theta$ can be found by the use of a table of natural tangents. Plot a curve, the abscissae of which are the values of D, and the ordinates the corresponding values of $\tan \theta$. This curve may now be used to find the value of $\tan \theta$, which is proportional to M, corresponding to any scale reading D.

Obtain from the curve the values of *tan θ* corresponding to all of the actual scale readings for the data recorded above. Plot the results for each run as ordinates against the corresponding currents producing the deflection.

Subtract from each ordinate of the curves for steel the ordinate corresponding to the same current for the coil alone. The differences for each curve form a second curve which represents the magnetic moment of the iron *alone*, for that case.

The numerical magnitude of any individual ordinate is of no interest, but the curves are of interest in representing the comparative behavior of the two specimens and coil.

Some very interesting conclusions can be drawn from the results and should be recorded in the final report.

53. ALTERNATING CURRENTS

Introductory Remarks. An *alternating current* is one which periodically reverses in direction a certain number of times per second. The time T during which the current completes one cycle of changes is called the *period*, and its reciprocal n is known as the *frequency* of the alternating current. The common commercial frequencies are 25 and 60 per second.

The almost continuously varying character of alternating currents causes effects which are not obtained in direct current circuits. Most of these effects are very useful and make possible, with alternating currents, many results not so easily accomplished with direct currents, such as easy transformation of voltage, etc. For this reason alternating current service for use in lighting and for power has practically replaced direct-current systems.

Although the phenomena obtainable with alternating currents are immensely more complicated than the steady current phenomena, and no single experiment can hope to present much of the theory of alternating currents, yet a few fundamental facts can be illustrated.

Object. The object of this experiment is to teach the fundamental difference between direct currents and alternating currents, and to show the effect of a change in frequency on the apparent resistance of a coil of wire. The effect on the apparent resistance of introducing an iron core into the coil will also be observed.

Apparatus. The alternating current for this experiment is derived from a machine which converts a direct current into an alternating current, and is sometimes known as an *inverted rotary converter*. The device is simply a direct-current motor equipped with slip rings which connect into the motor winding on the armature. The speed of the machine can be varied, by means of a field rheostat, from a few revolutions per second to about 60. The frequency of the resulting alternating current is the same as the speed per second.

A coil of several hundred turns of wire wound on a wooden spool is fed with current by the converter. A core of iron wires and a copper tube, which fits over the core, can both be inserted into the coil. An adjustable resistance is provided to be connected in series with the coil.

An alternating current ammeter and voltmeter; known as *hot-wire* instruments, are provided for indicating the current and voltage supplied to the coil. These instruments depend in their action upon the expansion of a fine wire due to the heating effect of the measured current. The ammeter does not read amperes directly, but the true current in amperes can be obtained from the posted calibration curve of the instrument.

A speed indicator is provided for determining the speed of the machine.

Discussion. If an alternating electromotive force of effective value E (reading of voltmeter) is impressed on a circuit containing a resistance R and a coil of wire, then the effective current (ammeter reading) is given by the expression

$$I = \frac{E}{\sqrt{R^2 + 4\pi^2 n^2 L^2}},$$

where n is the frequency per second, and L is a new property of the coil called *inductance*. It is in virtue of this inductance that there is produced in the coil an opposing electromotive force whenever the current changes, which depends upon the rate of change of the current. This opposing electromotive force and the ohmic resistance R of the circuit both serve to reduce the current. If L is zero, the above formula reduces to Ohm's law. The full formula may, therefore, be spoken of as the *extended* Ohm's law.

If the circuit contains no iron, the inductance L is constant and depends simply upon the geometry of the circuit. If iron is inserted in the coil, the inductance depends both upon the frequency and the current strength and is, therefore, *not* constant. The value of L in the formula represents, in this case, therefore, something analogous to the average inductance.

Procedure. Start the generator on low speed, and regulate the resistance in series with the coil so that the current is about as much as the instrument will read. Take readings of voltage, current, and speed as the frequency is increased from its lowest value to about 60 per second. The speed should be taken for at least a minute.

Repeat the same run with the iron core in the coil.

Again repeat the run with the copper tube over the iron core.

Obtain, from the calibration curve of the ammeter, the true currents corresponding to the ammeter readings.

Results. Plot $\frac{E}{I}$ as ordinates against n as abscissae for the three cases, and, from several points on each curve, calculate the value of the inductance L. Tabulate the results. The intercept on the E/I axis gives the resistance R.

Explain the effect of the copper tube.

SECTION V. LIGHT

60 PHOTOMETRY.

Introductory Remarks. *Photometry* is the science of the measurement of the intensity of light in terms of a standard unit of intensity, and the *photometer* is the device by means of which this measurement is accomplished.

Photometry has become a very common commercial process, and is of particular interest because of the practical problems with which it deals. Among the problems are the determination of the distribution of light around various light sources together with the effect on this distribution of shades of various shapes, and the comparative efficiencies of various kinds of light. Both problems will be partially investigated in the following experiment; suggestions will be given of further work which it would be interesting to carry out, if there is time.

The value of the following experiment lies in the practice gained in the use of a commercial photometer, in the practical knowledge of the distribution of intensity of the light about bare and shaded incandescent lamps, and in the efficiency of lamps at different voltages.

Object. The object of this experiment is to show the comparative behavior of two types of lamps at different voltages, and to measure the effect of a glass shade on the distribution of light around a lamp in a plane through the long axis of the lamp.

Apparatus and Discussion. A Lummer-Brodhun photometer is provided for use in this experiment. A description of the apparatus is given below.

THE LUMMER-BRODHUN PHOTOMETER AND PHOTOMETRY

There are many types of photometers of which the most common commercial forms depend upon the law of the inverse square of the distance. The two sources of light which are to be compared, the standard and the light under investigation, are placed at a considerable distance apart, and a screen moved until it is equally illuminated by each. The relative intensities of the two sources are then proportional to the squares of their distances from the screen. Of this type the Lummer-Brodhun form is a very satisfactory example. In this photometer the two sources of light are placed at opposite ends of a long scale on which slides the co-called *head* of the photometer. The head of the Lummer-Brodhun photometer is shown in Fig. 37. It consists of a white plaster of Paris disk S, both sides of which are viewed at the same time through the telescope T. This is accomplished by means of a system of mirrors, two of which are shown at M and M'. The light from the mirror M is reflected by the right-angle prism P into the telescope by total internal reflection from the back of the prism. There is a corresponding prism P'. The two prisms P and P' do not touch except over a certain area where they are cemented together by Canada balsam. Where the prisms do not touch, the light is entirely reflected by the prism P, and all the light received is from the mirror M and the side S of the screen. Where the prisms are cemented together by the Canada balsam no light is reflected, but the light is entirely transmitted from the prism P', and the light comes from the mirror M' and the side S' of the screen. The telescope is focused on the surface of separation of the two prisms, and the observer, thus seeing side by side the two surfaces S and S', can see when the two sides are equally illuminated. The head of the photometer is moved backward and forward until this is accomplished. The position of the head on the long scale is then read, and the relative intensities of the two sources calculated by means of the ratio of the squares of the distances.

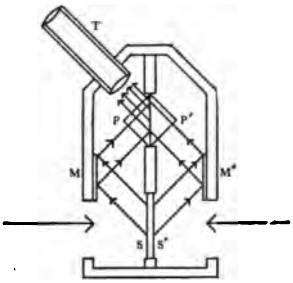

Fig. 37

The unit in terms of which the intensity of light is usually measured is that of a paraffin candle burning at the rate of 120 grains, 7.78 gms., per hour. It is, however, difficult to make a candle burn steadily, and it is now customary to use some other standard, the candle power of which has been determined.

The Hefner lamp is in common use as a standard because of its steadiness, and because the substance burned can easily be obtained pure. This lamp has a small round wick, and resembles an ordinary alcohol lamp except that it burns amyl acetate. If the wick has definite dimensions and the flame is adjusted, by means of a sighting device on the lamp, to a definite height, the candle power is known. The light of this standard has the disadvantage of being slightly reddish in color and hence difficult to accurately compare with white lights.

The usual practice in photometry is to use as a secondary standard an incandescent electric lamp, which has been carefully standardized against a Hefner lamp. This secondary standard is usually a specially constructed lamp with a filament in a single plane, and the lamp is placed in the photometer so that this plane is perpendicular to the direction of the photometer scale.

The standard lamp for use with this experiment has been carefully standardized and the candle power marked on the lamp.

The particular photometer used in this experiment is so arranged that the distance between the photometer head and the standard lamp can be kept constant. This is not necessary but considerably simplifies the computation.

Means are provided on the photometer for tipping the test lamp through measured angles in a vertical plane and for rotating it about a vertical axis through indicated angles. In this way the intensity of light can be measured in any direction. Since most shades are symmetrical about an axis, it is usually sufficient to know the distribution of light in a vertical section through the axis of symmetry.

Carbon and metallized carbon are now almost entirely replaced by the recently developed, and much more efficient, metal filament lamps. It is interesting to know just how much superior the new lamps are to the old in light-giving power, and in just what particulars the two kinds of lamps differ. This experiment includes tests designed to investigate the important characteristics of the lamps, and their behavior under various conditions.

An incandescent lamp receives from the mains a certain amount of electrical energy per unit time, a small part of which is transformed into useful light, the remainder going over into useless heat. The most interesting characteristic of a lamp is, therefore, its *light efficiency.* This might be stated, as is usually the case when stating efficiency, as its output in light divided by its input in electrical energy. If it were possible to determine the energy of the total light given out by the lamp, then the output and input could be expressed in the same units, and the efficiency would come out in per cent. This form of expressing efficiency is, however, not practicable because of the difficulty of determining the energy output of light, nor would the efficiency expressed in per cent be as useful to the engineer as the usual method of stating the performance of a lamp given below.

Commercially, the efficiency is stated in the following way. Suppose it be agreed that the candle power of the lamp be measured in a direction perpendicular to the long axis of the bulb, and that the mean horizontal candle power denote the average of the horizontal candle powers measured at regular angular intervals around the lamp in a plane perpendicular to its long axis. Then, the mean candle power, thus measured, divided into the input to the lamp expressed in watts, gives the watts per mean horizontal candle power which is the common manner of expressing the quality of a lamp. This measure of the lamp's light-giving power indicates the *inefficiency* rather than the efficiency of a lamp for emitting light.

Procedure. Adjust the standard lamp so that the plane of the filament is perpendicular to the axis of the photometer. Throw the double-throw switch over to the right; the voltmeter then reads the voltage on the standard lamp. A resistance is provided in series with the lamp by means of which this voltage may be regulated, and it should be kept throughout the experiment at 110 volts. It should be examined and if necessary regulated, at frequent intervals. On throwing the double-throw switch to the left the same voltmeter records the voltage on the test lamp, and another resistance is provided to vary this. An ammeter is connected in series with the test lamp.

Place the metallized carbon lamp in the test lamp socket, and, adjust the resistance so that the lamp is up to full brilliancy.

Set the lamp vertical and at some angle, say 0°, which is to remain fixed for the following run. Record the voltage and current for the test lamp, and the photometer readings for six or eight conditions of brilliancy from the maximum down to the lowest measurable candle power.

Replace the carbon lamp with a 25-watt tungsten lamp, and repeat the run as made with the carbon lamp.

Record the candle power of the standard, which is to be obtained from a chart furnished with the apparatus. The value is to be taken which corresponds to the voltage used.

Then, with the tungsten lamp in the test socket take readings of the photometer as the test lamp is turned through 180° by steps of about 15°, beginning with its tip pointing toward the standard lamp.

Attach around the test lamp the frosted glass shade and repeat the run.

Measure the distance between photometer head and the standard lamp.

Results. The ratio of the candle power of a lamp in the chosen fixed direction and the mean horizontal candle power is a constant for all brilliancies, and can be used for converting the candle power in the fixed direction for any brilliancy into the corresponding mean horizontal candle power. This factor is for lack of time here assumed to be unity, though an error of 10–15 % may thereby be introduced.

The *watts per mean horizontal candle power* and the *resistance* of the lamp are both to be calculated for each lamp, and plotted as ordinates against *candle power* as abscissae. The two curves are to be plotted on the same piece of paper in order that comparisons between the lamps may easily be made.

The two curves from data taken at the various angles, with bare lamp and with shade, are to be plotted on the same sheet of polar coördinate paper. The intensity of light is indicated by the distance from the center along the particular radius which corresponds to the direction in which the intensity was measured. Fig 38 shows a possible intensity curve about a shade.

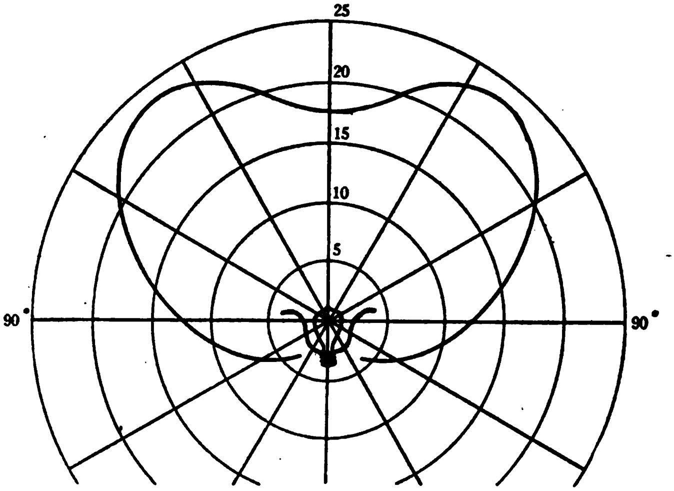

Fig. 38

Comments should be made as to the effect of the shade.

If time permits it is interesting to start with a lamp vertical at 0° and turn it around a vertical axis through small steps, thus testing the variation in horizontal candle power. An average of all such results gives the mean horizontal candle power and a true value for the ratio assumed above to be unity.

It is also worth while to examine a clear shade instead of a frosted one, and other shapes of shades.

Suggestion. Much time may be saved in the computation of this experiment if a slide rule or a table of squares be used.

INTRODUCTION TO EXPERIMENTS 61, 62 AND 63

Optical instruments of one kind or another are of such general use in everyday life as well as in practically every branch of science that even a small knowledge of their construction and theory is of great interest and practical value. The two following experiments are designed to supply some information concerning the more common optical instruments. In the experiment called *Optical Instruments,* several optical instruments will be constructed and their characteristics studied. The second experiment deals with the theory and use of one of the most useful of optical instruments, the compound microscope.

Optical instruments consist of various combinations of lenses, sometimes combined with prisms or other optical devices. A thorough knowledge of the action of lenses is required, and some text-book should be consulted if necessary. The two following sections on the lens formula and eyepieces should be read before attempting the experiments.

NOTE CONCERNING THE USE OF THE LENS FORMULA

Various conventions regarding the signs to be used in the familiar lens formula are found in text-books. In some books different formulae are used for the converging and diverging lenses, while in others one formula serves for both, with a convention regarding the proper signs to be used.

For the sake of uniformity in this course, the following convention is to be used.

The formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{F}$$

will serve for any case with any lens if the following simple rules be remembered.

1. The focal length, F, is *positive* when dealing with a *converging* lens, and *negative* when dealing with a *diverging* lens.
2. The distance, a, from object to lens is *positive* if the rays of light *diverge* as they enter the lens, as they do from a real object; a is *negative* if the rays of light *converge* as they enter the lens, as is sometimes the case when the light comes from another lens. The object in the last case is sometimes called virtual.
3. If the distance, b, from image to lens comes out *positive,* the image is *real;* if b is *negative,* the image is *virtual.*

Exactly the same formula and rules apply to mirrors.

EYEPIECES

Practically every optical instrument made for visual use has an eyepiece made up of two or more lenses. There are two common forms of eyepieces known as the Huygens' or negative eyepiece, and the Ramsden's or positive eyepiece.

The Huygens' or negative eyepiece is shown in Fig. 39, and consists of a field lens *F* and an eye lens *E*, their focal lengths being in the ratio of 3 to 1, respectively. The two lenses are placed twice the focal length of the eye lens from each other. The field lens is here shown as a double convex lens and such may be used in the experiments on optical instruments, but a plano-convex lens with its plane side toward the eye lens is generally used. The intersection of the dotted lines giving the position of the virtual image is not shown in the drawing.

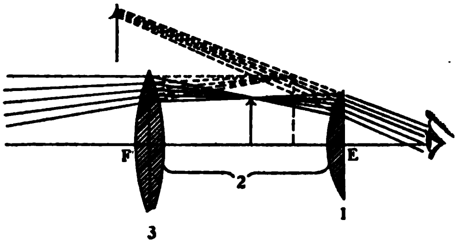

FIG. 39

The advantages of the negative eyepiece lie in its great field of view, in the remarkably small amount of both spherical and chromatic aberration, and on the flatness of the field.

The Ramsden's or positive eyepiece is shown in Fig. 40. The two lenses are of equal focal length, and are separated by a distance equal to 2/3 the focal length of either lens. The field lens is again shown as a double convex lens but more often a plano-convex lens is used with the convex side toward the eye lens. The final virtual image is too distant from the lenses to be conveniently shown on this drawing.

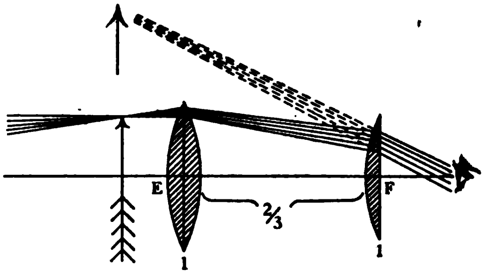

FIG. 40

The aberration is great in a positive eyepiece especially at the borders of the field, but the arrangement has the advantage of being more conveniently used with a micrometer, since the image and micrometer graduations, which it views directly, lie just outside the field lens, whereas in a negative eyepiece the image and micrometer graduations are between the lenses.

61. OPTICAL INSTRUMENTS

Read *Introduction to Experiments* 61, 62 *and* 63, the section on *Eye-pieces*, and the *Note on the Use of the Lens Formula.*

Object. To study the formation of images by a simple lens, and the action of two forms of telescopes.

Apparatus. Optical bench, with supports; a large simple lens to serve as objective lens; a one-inch achromatic objective lens; a small simple positive lens for eyepiece; a negative lens for the same purpose; electric light at some distance; spherometer and flat glass plate; screen with central hole to use with large lens; small green color screen.

Method. (*a*) Find the focal length of each of the three positive, or converging, lenses furnished for this experiment. To do this, mount the lens in one holder, and a clean white card in another. Turn the instrument toward a window, and obtain a sharp image of a distant object on the card. Measure the distance between the center of the lens and the card, which will be the desired focal length. In all cases set the lens perpendicular to the direction of the light.

Measure the radii of curvature of the large lens by means of the spherometer. This instrument is to have its zero reading determined by placing it on the flat glass plate, and setting all four feet in the same plane. Readings should be made to fractions of the smallest division on the head of the instrument, and more than one setting is necessary. Make an impression of the four feet on a piece of paper, from which the distances apart may later be measured. Then place the instrument on the spherical surface to be measured, and find the elevation (h) of the central foot, when all four feet are in contact with the surface.

Make a diagram representing a plane section of the lens, perpendicular to the lens surface at the center, and passing through one outer foot and the central foot of the spherometer. From the geometry of the arrangement, work out the relation between the radius of curvature of the lens, the quantity h and the distance from the center to the outer feet.

From the radii of curvature, work out the focal length of the lens, assuming an index of refraction of 1.55; compare this with the observed value.

(*b*) Make up a telescope, using the large simple lens as objective, and the small positive lens as eyepiece. Point this toward the electric lamp, and obtain as sharp an image as possible. Observe the character of this image with the eyepiece at various distances. Place the screen with central hole over the large lens, and note the change in the character of the best obtainable image. Turn the objective about a vertical axis, and observe the changes in the image; set the objective at the angle giving the best image. Insert, next the eye, holding it there by hand, the small green color screen, and record the change it produces in the best obtainable image.

(*c*) Replace the simple objective lens by the achromatic lens, and adjust this telescope to give a sharp image of the electric lamp (color screen removed). Measure the distance of the eyepiece from the image which is to be found between the two lenses, and from this distance and the focal lengths of the objective and the eyepiece, calculate the position of the final image, and the magnifying power of the telescope.

(*d*) Keeping the achromatic objective, replace the positive eyepiece by the negative one. Focus this telescope ("opera glass") on the electric lamp, and measure the distance apart of the centers of the two lenses. Comment on this type of telescope, as compared with that in (*c*).

(*e*) With the same lenses as in (*d*), set up a white screen some distance beyond the eyepiece, and adjust the lenses to act as a magnifying "telephoto lens" combination. When a good large image of the electric lamp is sharply focussed on the screen, measure all the distances that you need, and calculate from these the focal length of the negative lens. Calculate the latter also from the data in (*d*).

62. DEFECTS OF LENSES

Object. To study the common defects of lenses; chromatic and spherical aberration, and astigmatism.

Apparatus. An "optical bench" consisting of a wooden base, with meter bar, and supports for a lens and an eyepiece, the first of these movable, with a pointer; an electric light across the room; a wire gauze in a window; two diaphragms, and three little color screens made to fit into holders on the eyepiece.

Method. (*a*) *Spherical Aberration.* The rays passing near the rim of any positive lens converge to a point on the axis of the lens which is nearer to the lens than the focus of central rays. In order to compare different lenses in this respect, it is usual to define the *amount* of the spherical aberration as the ratio of the difference of focal length for central and marginal rays to the usual focal length. This quantity is to be measured.

Place in front of the lens the diaphragm which has a small central hole in it. The lens must be at right angles to the line of sight. Point the instrument toward the electric light, and focus it, as a telescope, so as to give as clear a view of the light as possible. Note the position on the meter bar of the marker on the lens holder, and repeat this setting several times, for accuracy. Then remove this diaphragm and substitute the other one, which permits marginal rays only to pass through. Repeat the adjustment of the lens support, as before. Find the change in focus in passing from one diaphragm to the other. Measure with a separate meter bar the distance from the lens to the image which it forms of the lamp, catching this on a card. This distance may be taken as the focal length with sufficient accuracy for the purposes of this experiment. Hence find the *amount* of the spherical aberration. Express this in per cent.

Note which surface of the lens faces the light; they are not alike. Then turn the lens around, so that the other side faces the light, and redetermine the amount of the spherical aberration. The fact that a different value results with lenses of certain shapes is important in the problem of eliminating spherical aberration in lens systems, as positive and negative lenses may be combined as pairs so that the aberrations are annulled through variations in shape, without at the same time destroying all refracting power in the combination.

(*b*) *Chromatic Aberration.* With the convex side of the lens facing the light, and the diaphragm with central hole in place, examine the changes of focal length when each of the three (red, green and blue) color screens are used, in succession. Find, from these observations the *amount* (defined as above) of the chromatic aberration for *red to green* and for *red to blue.*

(*c*) *Astigmatism.* Keep the lens as in (*b*), with green screen in place. Sight on an object consisting of horizontal and vertical lines (such as a piece of wire gauze in the window). Set the pointer of the little circular scale on the lens holder at zero, and adjust the lens to be at right angles to the line of sight at the same time; then obtain a sharp image, as seen through the eyepiece. Both horizontal and vertical lines will appear sharp at the same time. Note by repeated trials the position of the lens holder on the meter bar. Then turn the lens around a vertical axis by 5°, and find the two positions at which horizontal and vertical lines (now separately) appear sharp. Repeat at 10°, 15° and on both sides; also further out, if there is time. Find the ratio of the difference in focal length for horizontal and vertical lines to the focal length itself at each angle, averaging the observations taken at equal inclinations on the two sides. This may properly be called the amount of the astigmatism.

Plot curves showing how the difference in focal length for the two sets of lines varies with the angle, and also plot the actual changes in focal length for each set of lines.

63. THE COMPOUND MICROSCOPE

Read *Introduction to Experiments 61, 62 and 63.*

Object. The object of this experiment is to teach the construction and action of the optical system of a compound microscope when adjusted for visual use and for photographic use or projection. The equivalent focal lengths of the objective and eyepiece will be found, and the total magnification for the visual and photographic use of the instrument will be determined.

Apparatus. The apparatus provided for this experiment are a compound microscope, a stage micrometer which is a piece of glass upon which are ruled fine lines 0.1 m.m. and 0.01 m.m. apart, a micrometer and positive eyepiece, a support carrying a ground-glass screen and shading hood, and a 6-volt tungsten light for illumination.

The compound microscope consists essentially of two lens systems, namely: an *objective,* which is the lower combination of lenses, and the *eyepiece,* which is usually made up of two or more single lenses. These lens systems are mounted on a metal barrel which can be easily moved up or down by means of a slow motion screw for the purpose of focusing. A stage for supporting the object to be viewed, a mirror for concentrating light upon the object, and various adjusting devices constitute the rest of the instrument.

Discussion. Although the eyepiece and objective are made up of several lenses, the series of lenses composing each is equivalent, so far as position and size of image is concerned, to a *single* convex lens. The focal length of this simple or equivalent lens is the focal length of the combination of lenses in the eyepiece or objective to which it corresponds, and it is these *equivalent focal lengths* which are useful in the use of the instrument, and are to be obtained in the following experiment.

In the ordinary visual use of the compound microscope, the objective forms a magnified real image of the object just *inside* of the principal focus of the eyepiece, so that the latter gives a still further magnified but virtual image at the distance of clear vision from the eye.

When the miscroscope is used for photography or for projection, the second or final image must be real, and this is obtained by causing the image produced by the objective to fall a small distance *outside* of the principal focus of the eyepiece.

In Figs. 41 and 42, the equivalent lens systems of the microscope and the positions of the images are shown for both visual and photographic use of the instrument, respectively.

If the size of the object be known, and the exact lengths of the two images be measured, and further, if the positions of the images with respect to the object be determined, then, by using the familiar lens formula, the equivalent focal lengths f and F can readily be calculated.

Procedure. Place the stage micrometer on the stage of the microscope, and adjust the mirror of the instrument so that sufficient daylight is concentrated upon the object to make it plainly visible through the instrument. Adjust the small eyepiece up or down in the tube of the micrometer box until the cross-hairs are well in focus. Next focus the microscope for the stage micrometer by slowly turning first the large milled heads, which serve for coarse adjustment, and then finally the micrometer screw, which will be found at the top of the supporting post. This adjustment insures that the first image of the lines of the stage micrometer produced by the objective lens system is exactly in the plane of the cross-hairs situated just below the eyepiece.

Measure, by means of the micrometer, the distance between the images of two lines separated by seven or eight spaces, which are the magnified 0.1 m.m. spaces. This measurement should be made several times, and the average size of the image of one space obtained. Note that on some instruments the pitch of the micrometer screw is 0.5 m.m., but the head of the screw is divided into 100 parts. Also measure the distance the eyepiece is drawn out of the tube of the micrometer box.

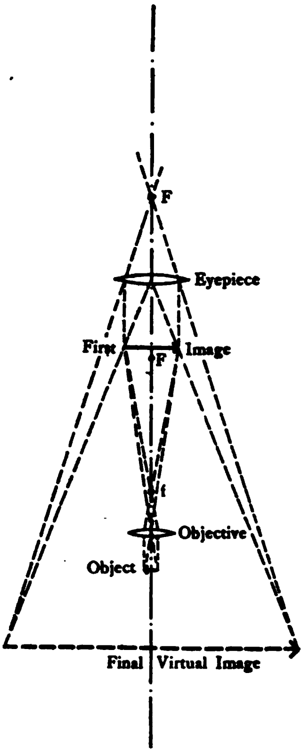

FIG. 41

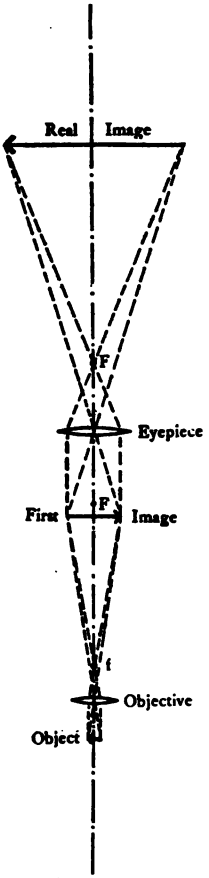

FIG. 42

Without disturbing the focus of the microscope, place the instrument under the conical cloth tube so that the small end of the cone hangs a centimeter or so below the eyepiece. Place the pasteboard collar around the eyepiece so as to obstruct any stray light from below from entering the hood. The ground-glass screen should be fixed in the holder with the ground side up. Swing the mirror of the microscope to one side, and place the tungsten light under the microscope stage so that the light passes straight up the tube.

No clear image will now be seen on the screen because the eyepiece is adjusted for visual use and therefore is set to give a virtual image. The eyepiece only is now to be moved until a sharp image of the stage micrometer and the cross-hairs appear upon the ground-glass screen. The width of one space as seen on the screen can now be determined by measuring the total width of a large number of spaces together and dividing by the number. This measurement should be made with a vernier gauge as accurately as possible.

The exact distance from the upper side of the stage micrometer glass to the upper side of the ground-glass screen is now to be measured. The distance from the object to the plane of the cross-hairs is also to be measured; this plane lies *1.5 mm.* below the flat top of the micrometer box. The new position of the eyepiece with respect to the tube should be recorded.

These data are sufficient to calculate the equivalent focal lengths of the objective and eyepiece.

Computation. Draw to 1/2 scale the equivalent lens system for the compound microscope when adjusted for photographic use, and indicate the position of the principal foci. Represent the object by an arrow about 1/16 inch long, and fill in all images to scale, indicating rays of light by the usual construction for obtaining an image.

From the observed distance through which the eyepiece must be moved downward in order to focus for visual use, calculate the position of the virtual image for such use, and draw another diagram similar to the first, giving the result in graphical form.

Results. The results required are as follows:

1. The equivalent focal lengths of the eyepiece and objective.
2. The two scale drawings as mentioned above, showing all images to scale.
3. The total magnification when adjusted for visual and photographic use.
4. The size and position of the virtual image when adjusted for visual use.

64. THE DIRECT-VISION SPECTROSCOPE

Introductory Remarks. The study of spectra is of great interest because of their wide theoretical and practical application. In many lines of theoretical research, spectra are either the direct phenomena studied, or they are necessary as experimental means in the study of other phenomena. The practical uses of spectra include qualitative spectrum analysis, and such special problems as the determination of the constitution of light coming from various artificial sources and the light transmitted through colored screens, in the attempt to combine artificial lights with color screens to imitate daylight, etc., etc.

SPECTRA

Continuous spectra are given by incandescent solids and liquids and, in general, give no means of telling the identity or nature of the incandescent substance other than its temperature. All gases and the vapors of the elements, and even some gases which are chemical compounds, give characteristic spectra when caused to emit light. These spectra are usually composed of bright lines or bands, and the fact that each substance has its own characteristic spectrum makes possible qualitative spectrum analysis.

When white light passes through a gas or vapor, those wave lengths of light are absorbed which are emitted when the gas or vapor is caused to give its spectrum. This absorption of definite wave lengths from the continuous spectrum of white light gives dark line spectra known as *absorption spectra.*

All transparent liquids and solids absorb certain portions of the spectrum, and it is this absorption which accounts for their color.

If the salt of a metal be vaporized in a Bunsen flame, a characteristic spectrum of the metal will in general be emitted. If the metal be made the terminals of an electric arc, a characteristic spectrum of the metal will be emitted, which spectrum may differ in some respects from the flame spectrum. Similarly, the spectrum emitted when the metal is made the terminals of an electric spark-gap, across which the discharge of a Leyden jar or condenser be made to pass, is characteristic of the metal, but is usually different from either the flame or arc spectrum. The mode of excitation must, therefore, always be stated.

The spectrum of a gas is usually obtained by causing a current of electricity under high voltage to pass through a glass tube containing the gas at low pressure.

Object. The following experiment seeks to give some familiarity with the different types of spectra, and to give experience in the use of the direct-vision spectroscope, an instrument for the easy observation and measurement of spectra. The arbitrary scale of the spectroscope will be calibrated in terms of wave lengths, and the instrument will then be used to determine the wave lengths of the principal lines in the spectra of several elements. The spectra of the several elements thus determined will be mapped out to scale, and the wave lengths of their principal lines determined.

Apparatus. The direct-vision spectroscope, shown in Fig. 43, is a convenient instrument for observing and comparing spectra, but it is not an instrument for accurate quantitative work. The instrument consists essentially of a series of prisms, alternately of flint and crown glass, so proportioned and placed that there is great dispersion of the light which passes through them with no net refraction of the middle of the spectrum. A slit at one end of the tube carrying the prisms can be adjusted in width by means of a milled nut. At the other end of the main tube in some instruments there is a disk carrying a series of lenses of various focal lengths so arranged that they can be singly adjusted in line with the prisms. This arrangement is provided for focusing instead of the usual sliding eyepiece. A side tube contains a lens, mirror, and graduated scale so arranged that light entering the end of the side tube projects the graduated scale so that it is superposed upon the spectrum which is being observed. By means of this scale, arbitrary readings can be taken of the lines in a spectrum, and, if the scale has once been calibrated in terms of wave lengths, the readings give a rough quantitative measure of the wave lengths of the lines being observed.

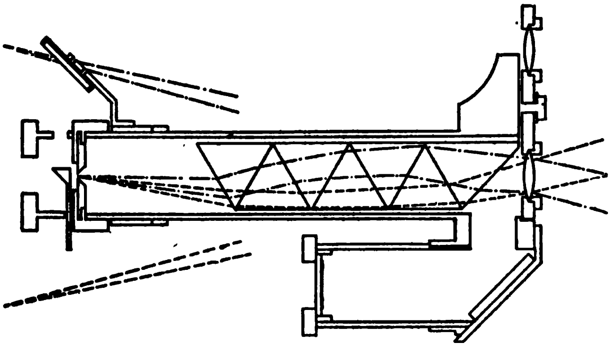
FIG. 43

In order to calibrate the scale of the spectroscope, some spectrum must be observed the wave lengths of the lines of which are known. For this purpose the cadmium spark spectrum will be used. A transformer, which boosts the voltage of the lighting mains from 110 to several thousand, charges a set of Leyden jars which, in turn, discharge across the metal spark gap. Great care should be exercised not to touch the electrical system while it is operating, as the shock might be serious.

Procedure. First focus very exactly the spectroscope for the *scale* by altering the eyepiece until a perfectly clear and sharply defined image of the scale is obtained. This adjustment may well be done with the slit closed. Now, by turning the milled head at the end of the instrument, make the slit very narrow. Point the instrument toward the sky, and focus for the slit, if necessary, by pulling the tube in or out until the fine dark lines or Fraunhofer lines are clearly seen and there is *no* parallax, or shift between cross-hairs and spectrum lines, when the eye is moved from side to side. The adjustments for focus outlined above should be done with great care.

Observe accurately the scale readings for the brightest lines of the cadmium spectrum, and identify the observed lines with those shown on the chart of the cadmium spectrum. The wave lengths of the principal lines given on the chart should be recorded.

While observing the spark spectra, it is well to place a piece of glass between the spark and the observer in order to obstruct the ultra-violet light which might otherwise burn the skin and eyes. The room should be well ventilated to carry off the metallic vapors.

Examine and record the readings corresponding to the brightest lines of the spectra of some other metals such as zinc, magnesium, copper, etc.

Results. Record the readings which mark the boundaries of the light and dark regions in the absorption spectra of two or three of the solutions provided. Note the effect of change in thickness of the solution of chlorophyll. Use a small tungsten lamp to give the continuous spectrum.

Plot a curve for which the scale readings for the cadmium spark are the abscissae, and the corresponding wave lengths of the cadmium lines, as given by the chart, are the ordinates. This is the calibration curve of the particular instrument used, and may now be used to determine the wave length from the scale reading for any unknown spectral line.

In the first or lower, horizontal, one-centimeter-wide space on the plotting paper, plot, as shown in Fig. 44, the principal lines of the cadmium spectrum.

In the 2d, 3d, 4th, etc., one-centimeter-wide strips, map to scale all the other spectra observed. By following up from any spectral line to the calibration curve, the wave length of each line can be read off. In this manner tabulate the wave-lengths of the principal lines in the spectra which are observed.

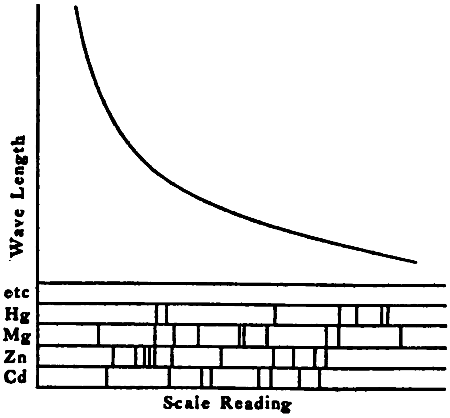

FIG. 44

INTRODUCTION TO EXPERIMENTS 65 AND 66

The two following experiments make use of the spectrometer, an instrument which, in some one of its various forms, is very commonly used in physical measurements and investigation. The experiments on the spectrometer are of value, first, in giving practice in the use of an instrument capable of great accuracy, and second, because of the important physical principles and phenomena with which they deal.

The following remarks are devoted to the explanation of the construction and use of the spectrometer.

THE SPECTROMETER

The spectrometer, shown in Fig. 45, is an apparatus for accurately measuring the angle between two beams of parallel light. The instrument is sometimes used to determine the index of refraction of a transparent substance in the shape of a prism from the measurements of its angles and the angle of minimum deviation of the light passing through the prism. The spectrometer is also commonly used in conjunction with a diffraction grating in determining the wave length of the light corresponding to different colors. The two following experiments illustrate the two uses of the spectrometer given above.

Of the two telescopes belonging to a spectrometer, one, called the *collimator*, is for rendering parallel the rays of light; the other is the *observing telescope*.

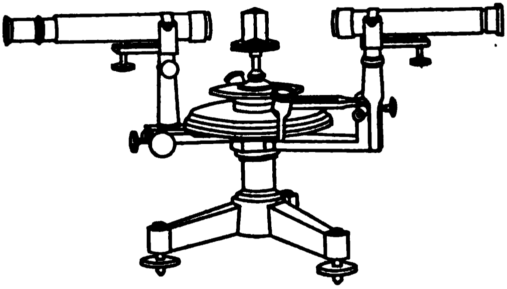

FIG. 45

The collimator has an adjustable slit at its outer end, which, when properly adjusted, is at the principal focus of the corrected lens at the inner end of the tube. The telescope is provided with a positive eyepiece and some cross-hairs which are viewed by the eyepiece. The spectrometer has a finely graduated circle and verniers (see page 13) for either indicating the angle through which the telescope is turned, or the angle through which the spectrometer table is rotated. The graduated circle, the most delicate and valuable part of the instrument, should be treated with special care.

Adjustments. The following adjustments are necessary in focusing the instrument. Focus the cross-hairs in the observing telescope until they can be distinctly seen without straining the eyes. Focus the telescope on some distant object, and adjust it until the object and cross-hairs can at the same time be distinctly seen, and until there is no parallax between them. Turn the collimator and telescope until they point directly toward the center of the instrument, and clamp firmly. Point the collimator toward the mercury lamp, turn the observing telescope about the central axis until it is opposite the collimator, and alter the length of the latter until the image of the slit, as seen in the observing telescope, is in sharp focus and without parallax with the cross-hairs; that is, until there is no shift between cross-hairs and slit when the eye is moved across the opening in the eyepiece.

The following further adjustments and precautions are necessary in the use of the spectrometer. Level the divided circle by means of a spirit level, and adjust the collimator and telescope so that they are level. If the latter adjustment is correct, the image of the slit will fall in the center of the field as seen through the telescope when the telescope and collimator are exactly opposite each other. The slit of the collimator should be narrow and quite vertical.

Throughout all work with the spectrometer, neither collimator nor telescope should be touched, and in turning the telescope its solid lower support should be grasped rather than the telescope itself, for otherwise the instrument might be thrown out of adjustment.

65. ANGLES OF A PRISM AND INDEX OF REFRACTION BY THE SPECTROMETER

Read *Introduction to Experiments 65 and 66.*

Object. The object of this experiment is to give practice in the use of the spectrometer when applied (1) to the accurate measurement of the angle of a prism, and (2) to the determination of the index of refraction of glass for the various colored lights given by the mercury vapor lamp.

Apparatus. A spectrometer, a dense glass prism, and a mercury vapor lamp are provided for this experiment.

The Mercury Vapor Lamp

The mercury vapor lamp is used as the source of light and consists of a long glass tube into the ends of which platinum terminals are sealed. Two pools of mercury, contained in pockets at the ends of the tube, make connection with these terminals and serve as electrodes. The tube is highly exhausted. When the tube is tipped until a continuous stream of mercury connects the two terminals, the circuit is completed and a current flows from the direct current lighting mains through a resistance and the lamp. When this thread of mercury breaks, an arc is formed which then spreads throughout the tube, giving the familiar greenish blue light. The mercury in the cups should never be allowed to boil away enough to appreciably lower the level in the cups.

Discussion and Procedure for (1) the Measurement of the Angle of a Prism. Focus and adjust the spectrometer as directed in the introductory remarks. Adjust the prism so that its edge is vertical as follows: Before placing the prism on the central stand, view the image of the slit when the telescope is opposite the collimator, and note its position vertically with reference to the top and bottom of the field of view, or with reference to the horizontal cross-hair. Place the prism on the stand with the edge formed by the two polished surfaces turned toward the collimator, and turn the observing telescope until the image of the slit can be seen by reflection from one face of the prism. Level the prism until the image of the slit occupies the same part of the field of view as when seen directly. Repeat, when viewing the slit reflected from the other face, and then, turning the telescope back, see that the result of the first leveling has not been disturbed.

There are two methods of determining the angle of a prism. Measure the angle of the prism by one of these methods, preferably the second. If time permits after the second part of the experiment, the same angles should be measured by the other method.

First Method for Measuring the Angle of a Prism. Place the prism at the center of the spectrometer with the edge formed by the two polished faces turned toward the collimator as shown on Fig. 46. Clamp the collimator and the prism table but leave the telescope free. Set the cross-hairs of the observing telescope upon the image of the slit reflected from one face of the prism, and read the graduated circle. The fine adjustment afforded by the tangent screw may be used for the final setting. Turn the telescope until the cross-hairs are set upon the image of the slit reflected from the other face. Measure the angle through which the telescope has been turned. It will be readily seen, by referring to Fig. 46, that this angle is twice that of the prism. Care must be taken not to confuse the reflected light with the refracted light which has passed around through the glass.

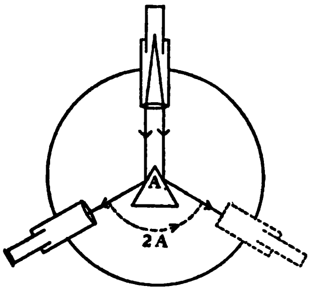

Fig. 46

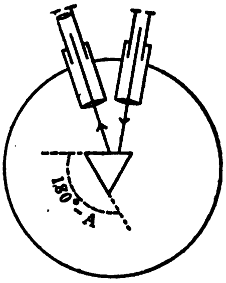

Fig. 47

Second Method for Measuring the Angle of a Prism. Clamp the observing telescope permanently near the collimating telescope as shown in Fig. 47. Place the prism upon the leveling base so that the image of the slit will be thrown into the observing telescope. Without disturbing either telescope, turn the prism by means of the tangent screw until the image coincides with the vertical cross-hair. Clamp the prism table, but leave the collimator clamping-screw loose. Take the reading on the graduated circle. Turn the *prism* until the image of the slit is reflected from the next face. The circle will turn with it, but the verniers remain fixed. Adjust accurately by the tangent screw, and again read. The angle through which the prism has been moved, subtracted from 180°, will be the angle A of the prism.

Discussion and Procedure for (2) Measuring the Index of Refraction. When a ray of light passes from a rare to a denser medium it is bent toward the perpendicular to the surface. For the same substances and the same colored light, the sine of the angle of incidence divided by the sine of the angle of refraction is a constant. It is called the *index of refraction*, and is usually denoted by the letter n. The index of refraction of light passing from air to glass may be most conveniently determined, when the glass is in the form of a prism, as follows:

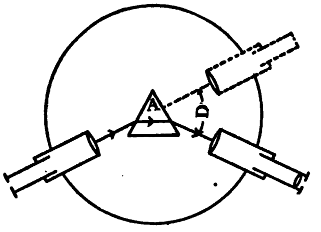

FIG. 48

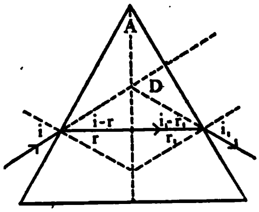

FIG. 49

Having completed the adjustments outlined above, turn the prism into the position indicated in Fig. 48. Set the observing telescope upon the green refracted image. Rotate the prism, following the refracted ray with the observing telescope. A position can be found for which the light is least refracted or least deviated from the direction of the collimator. Clamp the support of the prism. Set the cross-hairs on the green image of the slit, and read the position of the telescope. Remove the prism and set the telescope upon the image of the slit seen directly. The angle through which the telescope has been turned is the angle of deviation of the light, in this case the angle of *minimum deviation D.*

In the position of minimum deviation, the light passes through the prism in a direction such that it makes equal angles with the surface of the glass in entering and in leaving.

$$i = i_1 \text{ and } r = r_1 \quad \text{See Fig. 49.}$$

Consider the small triangle formed by the three paths of the ray extended. From the property of the exterior angle of a triangle

$$D = (i - r) + (i_1 - r_1) = 2i - 2r.$$

From the property of similar triangles, each half of A is equal to r.

$$A = 2r \text{ or } r = \frac{A}{2},$$

and substituting

$$D = 2i - A,$$

$$i = \frac{D + A}{2}.$$

Since

$$n = \frac{\sin i}{\sin r}$$

$$n = \frac{\sin \dfrac{D + A}{2}}{\sin \dfrac{A}{2}}.$$

Determine, by the method outlined above, the index of refraction of the prism for each of the five bright lines of the mercury arc spectrum. The five brightest mercury lines comprise two orange-yellow lines close together, a bright green line, a blue-violet line, and a deep violet line. There are other less distinct lines in the mercury spectrum, the index of refraction of which may be measured if time permits.

Results. The results to be derived are as follows:

1. Angle of prism by 1st method or by 2d method.
2. Index of refraction for the bright lines in the mercury spectrum.

66. WAVE LENGTH OF LIGHT BY MEANS OF THE DIFFRACTION GRATING AND SPECTROMETER

Read *Introduction to Experiments 65 and 66.*

Object. The purpose of this experiment is to give experience in the use of the spectrometer, and to illustrate the common method of measuring the wave length of light. The diffraction grating will be used to determine the wave length of four of the five bright visible lines of the mercury arc spectrum. In the case of the fifth or green line, the problem will be reversed, and the number of rulings per centimeter on the grating will be determined from the known wave length of the green line. This work with the green line will be done first, and the number of rulings thus obtained is to be used in the calculation of the wave lengths of the other four lines as stated above.

Apparatus. A spectrometer and a good grade of diffraction grating will be used in conjunction with a mercury vapor lamp as the source of light.

The surface of the diffraction grating should under no condition be touched, as the fine rulings are very delicate and would be injured.

Discussion. The best method of measuring the wave length of light is by means of the diffraction grating mounted on a spectrometer. The method was devised by Fraunhofer, who at first used a grating made of parallel wires. Crude as the device was, his determinations of the wave length of sodium light were surprisingly accurate. The modern diffraction grating is made by diamond rulings on glass or speculum metal.

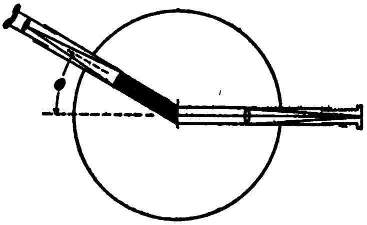

FIG. 50

The arrangement of the apparatus for a transmission grating on glass is shown in Fig. 50. The series of dots between the collimator and the observing telescope indicates the grating seen in cross section. The grating is placed on the central stand, with its plane at right angles to the axis of the collimator and with its rulings vertical. A series of spectra are thus produced, lying both to the right and to the left of the direct image of the slit. The explanation of the formation of these spectra is as follows:

The light passing through the narrow space between the rulings spreads laterally, but is of feeble intensity except in such directions that the light coming from all the openings are in phase with each other, and therefore additive in their total effect. Thus, in Fig. 51, which shows diffraction grating very much enlarged, if the angle with the original direction of the light is such that the path of each ray is longer than the one next to it by exactly one wave length, the light coming from the several openings will be in the same phase. In this direction the light will be intense and comparable with, though not equal in intensity to, the light seen directly through the grating. The same would be true with a difference of path of two, three, or any whole number of wave lengths.

Since the direction of maximum intensity depends upon the wave length, the light from the slit passing through the grating will be spread out into spectra. The first order spectrum is the spectrum nearest the direct image of the slit on each side, and is due to a difference in path for the successive slits of a single wave length. The second and third order spectra are located farther from the central image and are less distinct. They are due to successive path difference of two and three wave lengths, respectively, and show correspondingly greater dispersion. It may happen that the violet end of a higher order spectrum may be superposed upon a spectrum of a lower order.

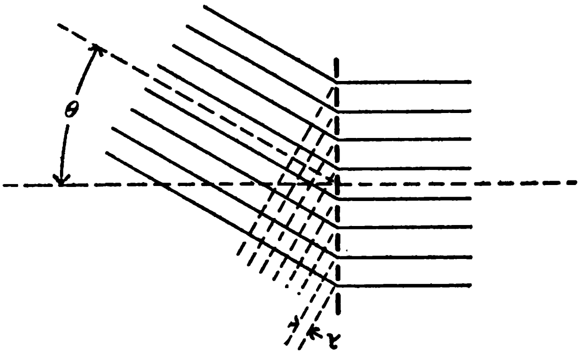

FIG. 51

Referring to Fig. 51 it will be seen that for the first order spectra

$$\lambda = d \sin \theta$$
$$= \frac{\sin \theta}{N},$$

where λ is the wave length of light for the particular line measured, d is the distance between rulings on the grating, N is the number of rulings per centimeter, and θ is the angle of diffraction.

For the second order spectra the formula becomes

$$\lambda = \frac{d}{2} \sin \theta = \frac{\sin \theta}{2N}.$$

Procedure. Focus and adjust the spectrometer as directed in the section on the *Spectrometer*.

Place the grating in the holder with its plane perpendicular to the axis of the collimator, and its lines exactly parallel to the slit. Measure the angle of diffraction in the spectra of as many orders as possible for at least three of the bright lines in the mercury spectrum. The measurements for the green line should be made first, and with special care, since an error in these measurements will decrease the accuracy of the remaining work. It would be well to make several independent settings for each case. Do more spectrum lines if there is time.

Record the number of rulings if it is given on the grating.

RESULTS. From the data for the green line, preferably in the second order, and its wave length, which is .00005461 cms., calculate the number of rulings N per centimeter of the grating. This value of N given on the grating is in some cases incorrect and will not, therefore, necessarily agree with the value obtained by experiment.

Using the calculated value of N, determine the wave lengths of the other four lines of the mercury spectrum, giving the results separately for each order.

67. ROTATION OF THE PLANE OF POLARIZATION OF LIGHT

Introductory Remarks. One of the most satisfactory methods of producing plane polarized light is by means of the Nicol prism. The Nicol prism is made of a rhomb of Iceland spar, cut in two at a suitable angle, and recemented with Canada balsam. This method of producing polarized light will be used in the following experiment.

Many substances, notably quartz, solutions of the sugars and the essential oils, have the peculiar property of rotating the plane of polarization of light. In some cases this rotation is to the left as the light comes toward the observer, and in other cases toward the right. In every case, however, the *specific rotation*, which is the angular rotation under standard conditions of distance traversed in the substance by the light, of concentration of solution, and of temperature and wave length, is a perfectly definite quantity for each substance.

The most important commercial application of this fact is in the analysis of various commercial products in order to find the amount of optically active constituent present. The analysis of the sugars and the products made of sugar are the most extensive analyses of this kind. The government regularly test in this way the importations of raw sugars in order to ascertain the duty which shall be imposed.

Object. The following experiment is chiefly concerned with the study of polarized light, and the effect of some substances in rotating the plane of polarization, but, in order to enforce the commercial application of the purely physical principle, some raw and refined cane sugars will be analysed in just the same way as is done at the custom house. The process is called *saccharimetry*.

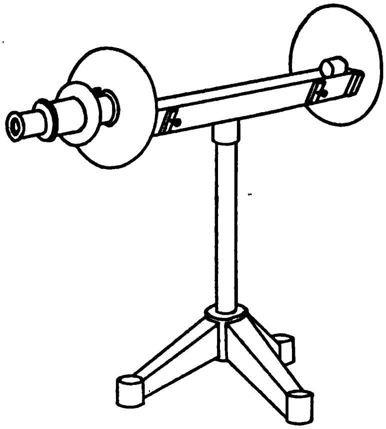
Fig. 52

Apparatus. The more elaborate apparatus for testing the amount of rotation of polarization is usually called a *polariscope*, or, if applied solely to the analysis of sugar, a *saccharimeter*. The simple arrangement used in this experiment consists of two Nicol prisms mounted at the ends of a brass-cradle as shown in Fig. 52. One Nicol is fixed and is called the *polarizer*. The other Nicol is capable of rotation through angles indicated on an attached scale, and is called the *analyser*. The cradle between the Nicols is arranged to receive glass tubes about 8 inches long containing the solutions to be tested, the direction of sight being through the tube lengthwise. Such an arrangement comprises a very simple polariscope.

In addition to the simple polariscope and solution tubes, glass flasks, funnels and some filter papers are provided to be used in mixing the solution.

Discussion. The angular rotation produced by the optically active substance or solution is directly proportional to the length of light path through the solution, almost directly proportional to the concentration of the solution, and somewhat dependent upon the temperature and the wave length of the light used. *The specific rotation is defined as the angle of rotation, expressed in angular degrees, which plane polarized light, corresponding in wave length to that of the yellow sodium line or D line of the solar spectrum, undergoes in passing through a decimeter column of a solution of the optically active substance at a temperature of* 20° *C., having a concentration of one gram in one cubic centimeter of solution.* The following formula expresses the facts given above:

$A^D_{20°} = (a)^D_{20°} \frac{lw}{v}$, where $A^D_{20°}$ is the measured angle of rotation at 20°C., $(a)^D_{20°}$ is the specific rotation as defined above, l is the length of column in decimeters, and w/v the concentration, i. e., the number of grams of substance w in v cubic centimeters of solution. If the experiment is not conducted at 20°C. but at t°C., then

$$A_t = A_{20°} - .011\ (t - 20)\ \frac{wl}{v}\,.$$

In the case of sucrose, which is the optically active substance in cane sugar, the specific rotation for the D line or sodium light, and at 20°C., is

$$(a)^D_{20°} = 66.7°\,.$$

This expression neglects a small correction dependent upon the concentration of the solution.

Combining these formulae, the expression for the angle of rotation for sodium light of a 100% solution of sucrose of concentration w/v at t°C. contained in a tube of inside length l decimeters is

$$A^D_t = \left[66.7 - .011\ (t - 20)\right] \frac{wl}{v}\,.$$

Comparing this with the actual measured angle of rotation, the result is the percentage purity of the substance being tested.

Procedure. Weigh out 30 grams of white granulated sugar, and put in one of the graduated 100 c.c. flasks. Fill up exactly to the mark with water, and dissolve all of the sugar. Filter the solution and fill one of the testing tubes. Using sodium light, determine the angle for total extinction of the light when the sugar tube is not in place. Make at least *ten* determinations of this reading. Place the sugar tube between the Nicols, and determine the reading for total extinction, making ten readings as before.

In the same way weigh out 15 grams of the raw brown sugar and dissolve in about 50 or 60 c.c. of water in a graduated 100 c.c. flask. Do not yet fill the flask to the mark. The solution must be clarified before it can be used in the polariscope. To do this, add about two cubic centimeters of the subacetate of lead solution, which will precipitate the albuminoids and impurities. In order to neutralize the possible excess of lead which would cause a cloudiness to appear after filtering, and to assist clarification, add, after shaking well, one or two cubic centimeters of the *alumina mixture*, which is aluminum hydrate held in suspension in ammonium sulphate. Now fill the flask up to the mark with water, and filter. Throw away the first few drops which come through the filter because they will be cloudy. Test the sugar in the same way described above.

Results. Calculate from the data the percentage purity of each grade of sugar.

APPENDIX

TABLES

1. *Weight in Grams of One Cubic Centimeter of Dry Air at Different Temperatures and Barometric Pressures*

Note. Two ciphers are to be prefixed to all the numbers in the body of the table.

Temperature (Cent.)	Barometric Pressure (Centimeters of Mercury)[1]							
	70 cm.	71 cm.	72 cm.	73 cm.	74 cm.	75 cm.	76 cm.	77 cm.
10°	.001149	1165	1181	1198	1214	1231	1247	1263
11°	1145	1161	1177	1194	1210	1226	1243	1259
12°	1141	1157	1173	1189	1206	1222	1238	1255
13°	1137	1153	1169	1185	1202	1218	1234	1250
14°	1133	1149	1165	1181	1197	1214	1230	1246
15°	1129	1145	1161	1177	1193	1209	1225	1242
16°	1125	1141	1157	1173	1189	1205	1221	1237
17°	1121	1137	1153	1169	1185	1201	1217	1233
18°	1117	1133	1149	1165	1181	1197	1213	1229
19°	1113	1129	1145	1161	1177	1193	1209	1224
20°	1110	1125	1141	1157	1173	1189	1204	1220
21°	1106	1121	1137	1153	1169	1185	1200	1216
22°	1102	1117	1133	1149	1165	1181	1196	1212
23°	1098	1114	1130	1145	1161	1177	1192	1208
24°	1094	1110	1126	1141	1157	1173	1188	1204
25°	1091	1107	1122	1138	1153	1169	1184	1200
26°	1088	1103	1118	1134	1149	1165	1180	1196
27°	1084	1099	1114	1130	1145	1161	1176	1192
28°	1080	1095	1110	1126	1142	1157	1172	1188
29°	1077	1091	1107	1122	1138	1153	1169	1184

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[1] In making barometer readings it is customary to read to the top of the curvature of the mercury, and this is sufficiently accurate for the purposes of the present course, but it is well to bear in mind that when greater accuracy is to be obtained corrections must be applied for the depression of capillarity, for the temperature of the mercury and of the measuring scale, and for the force of the earth's attraction at the particular locality in which the experiment is being performed.

2. Space in Cubic Centimeters Occupied by One Gram of Water at Different Temperatures

0°	1.000 12	10°	1.000 26	20°	1.001 73
1°	1.000 07	11°	1.000 35	21°	1.001 94
2°	1.000 03	12°	1.000 46	22°	1.002 16
3°	1.000 01	13°	1.000 57	23°	1.002 38
4°	1.000 00	14°	1.000 70	24°	1.002 62
5°	1.000 01	15°	1.000 85	25°	1.002 87
6°	1.000 03	16°	1.001 00	26°	1.003 13
7°	1.000 07	17°	1.001 16	27°	1.003 39
8°	1.000 12	18°	1.001 34	28°	1.003 67
9°	1.000 18	19°	1.001 53	29°	1.003 95

3. Vapor Tension of a 10% Solution of Sulphuric Acid and Water

Temperature	Vapor Tension in cm. of mercury	Temperature	Vapor Tension in cm. of mercury
15°	1.15	20°	1.58
16°	1.23	21°	1.67
17°	1.31	22°	1.77
18°	1.39	23°	1.88
19°	1.48	24°	1.99

PRINTED AT
THE HARVARD UNIVERSITY PRESS
CAMBRIDGE, MASS., U. S. A.